Lecture #12: Rice's Theorem and the Recursion Theorem
What Will Happen During the Lecture

Remember... You Had Homework!

Students were asked to have worked through a set of notes before this first lecture:

- Lecture Notes — "Rice's Theorem and the Recursion Theorem"

Of course, you may attend the lecture presentation if you have not worked through this material ahead of time — but it will not be repeated for you during the lecture presentation.

Problems To Be Solved

A Generalized Fixed Point Theorem

To begin, the following ***generalization*** of the "Fixed Point Theorem", which was included in the preparatory reading, will be discussed.

Theorem (Generalized Fixed Point Theorem)**.** *Let $f : \mathbb{N} \to \mathbb{N}$ be a Turing-computable total function and let k be a positive integer. Then there exists a number $e \in \mathbb{N}$ such that*

$$\Phi^{(k)}_{f(e)}(x_1, x_2, \ldots, x_k) = \Phi^{(k)}_e(x_1, x_2, \ldots, x_k)$$

for all $x_1, x_2, \ldots, x_k \in \mathbb{N}$ — that is, the $\mathcal{S}$-program with encoding $f(e)$ computes the same k-ary partial or total function as the $\mathcal{S}$-program with encoding e.

Application: Proving That a Function is Computable

This will be used to prove that a function, $f : \mathbb{N}^2 \to \mathbb{N}$, satisfying the following equations, is computable.

(a) $f(x, 0) = x + 2$ for all $x \in \mathbb{N}$.

(b) $f(x, 1) = 2 \times f(x, 2x)$ for all $x \in \mathbb{N}$.

(c) $f(x, 2t + 2) = 3 \times f(x, 2t)$ for all $x, t \in \mathbb{N}$.

(d) $f(x, 2t + 3) = 4 \times f(x, 2t + 1)$ for all $x, t \in \mathbb{N}$.

This is intended to provide evidence that the "Recursion Theorem" really *was* significant (for its time, and place) — and useful for showing that recursively defined functions are computable.