Lecture #15: More about Reductions and Reducibilities
Exercises and Review

Questions for Review

More about Reductions and Reducibilities

1. Suppose $A, B \subseteq \mathbb{N}$.

   (a) Say what it means for A to be ***many-one reducible*** to B.
   (b) Say what it means for A to be ***one-one reducible*** to B.
   (c) Say what it means for A to be ***Turing reducible*** to B.

   Note that the set of all many-one reductions, the set of all one-one reductions, and the set of all Turing reductions are all examples of reducibilities.

2. How are these three reducibilities related? For example, if $A, B \subseteq \mathbb{N}$ and $A \preceq_T B$ then is it always true that $A \preceq_m B$ too?

3. Suppose that $\preceq_Q$ is a reducibility. Let $A, B \subseteq \mathbb{N}$. What does it mean for A and B to be ***Q-equivalent***?

4. Let $\mathcal{W}$ be a collection of subsets of $\mathbb{N}$ and let $\preceq_Q$ be a reducibility. What does it mean for $\mathcal{W}$ to be ***Q-closed***?

5. If $\preceq_Q$ and $\mathcal{W}$ are as above, what does it mean for a subset A of $\mathbb{N}$ to be ***Q-complete for $\mathcal{W}$***?

6. Describe a process — which generalizes something you learned to do in CPSC 413 — to prove that a given subset $B \subseteq \mathbb{N}$ is Q-complete for a collection $\mathcal{W}$, for a reducibility $\preceq_Q$.

7. What is ***co-$\mathcal{W}$***, when $\mathcal{W}$ is a collection of subsets of $\mathcal{N}$?

8. Sketch a proof that if $\boldsymbol{W}$ is 1-closed (respectively, m-closed and T-closed) then so is co-$\mathcal{W}$.

9. Is the set of all ***computable sets*** 1-closed? ... m-closed? ... T-closed? *Briefly* say how this is proved.

10. Is the set of all ***computably enumerable sets*** 1-closed? ... m-closed? ... T-closed? *Briefly* say how this is proved.
11. Name and describe a specific subset of $\mathbb{N}$ that is 1-complete for the class of all recursive sets.
12. Suppose that $A \subseteq \mathbb{N}$ and that A is m-complete for the class of all computably enumerable sets. What else can be proved about A?
13. Suppose that $A, B \subseteq \mathbb{N}$. What is the set $A \oplus B$? How is it related to A and B?

Sets That are "Computably Enumerable, Relative to an Oracle"

14. Suppose that $G : \mathbb{N}^k \to \mathbb{N}$ is a ***total function*** for some integer $k \geq 1$ and that $B \subseteq \mathbb{N}$.

    (a) What does it mean for B to be ***G-computable***?

    (b) What does it mean for B to be ***G-computably enumerable***?

    How are these related to the concepts of being "computable" and "computably enumerable," respectively?
15. List two (or three) results concerning computable and computably enumerable sets that ***relativize***, so that they also hold for G-computable and G-computably enumerable sets, when G is as above.
16. State two other conditions that are satisfied by a set $B \subseteq \mathbb{N}$ if and only if B is G-computably enumerable.
17. What is the set W_n^G? State the ***Relativized Enumeration Theorem***.

Problems To Be Solved

1. Prove that if $A, B \subseteq \mathbb{N}$ and $A \preceq_T B$ then $\overline{A} \preceq_T \overline{B}$ as well.