

Lecture #16: The Arithmetic Hierarchy

Lecture Presentation

Closure Properties for Σ_n

Closure Under Union

Question: Let $n \in \mathbb{N}$ and let $S, T \subseteq \mathbb{N}$. Is it true that if $S, T \in \Sigma_n$ then $S \cup T \in \Sigma_n$ as well?

Answer:

Proof:

Closure Under Intersection

Question: Let $n \in \mathbb{N}$ and let $S, T \subseteq \mathbb{N}$. Is it true that if $S, T \in \Sigma_n$ then $S \cap T \in \Sigma_n$ as well?

Answer:

Proof:

Closure Under Another Operation

Question: Let $n \in \mathbb{N}$ and let $S, T \subseteq \mathbb{N}$. Is it true that if $S, T \in \Sigma_n$ then $S \setminus T \in \Sigma_n$ as well?

Answer:

Proof:

Closure Properties for Π_n

Let n be a **positive** integer and let $S, T \subseteq \mathbb{N}$.

- (a) Is it true that if $S, T \subseteq \Pi_n$ then $S \cup T \in \Pi_n$?
- (b) Is it true that if $S, T \subseteq \Pi_n$ then $S \cap T \in \Pi_n$?
- (c) Is it true that if $S, T \subseteq \Pi_n$ then $S \setminus T \in \Pi_n$?

Answers and Proofs:

Closure Properties for Δ_n

Let n be a **positive** integer and let $S, T \subseteq \mathbb{N}$.

- (a) Is it true that if $S, T \subseteq \Delta_n$ then $S \cup T \in \Delta_n$?
- (b) Is it true that if $S, T \subseteq \Delta_n$ then $S \cap T \in \Delta_n$?
- (c) Is it true that if $S, T \subseteq \Delta_n$ then $S \setminus T \in \Delta_n$?

Answers and Proofs:

Closure Properties for $\mathcal{AH}$

Let $S, T \subseteq \mathbb{N}$.

- (a) Is it true that if $S, T \subseteq \mathcal{AH}$ then $S \cup T \in \mathcal{AH}$?
- (b) Is it true that if $S, T \subseteq \mathcal{AH}$ then $S \cap T \in \mathcal{AH}$?
- (c) Is it true that if $S, T \subseteq \mathcal{AH}$ then $S \setminus T \in \mathcal{AH}$?

Answers and Proofs:

