Lecture #17: Applying the Arithmetic Hierarchy
Exercises and Review

Questions for Review

1. Suppose $P : \mathbb{N}^s \to \{0, 1\}$ is a ***total*** predicate. What set $S_P \subseteq \mathbb{N}$ is associated with P? How are these sets related, if $s = 1$?
2. What does it mean for P to be a $\boldsymbol{\Sigma_n}$***-predicate***? ...a $\boldsymbol{\Pi_n}$***-predicate***? ...a $\boldsymbol{\Delta_n}$***-predicate***?
3. Describe several useful ***closure properties*** of Σ_n-predicates, Π_n-predicates, and Δ_n-predicates.
4. What is an $\exists^{\boldsymbol{n+1}}$***-predicate***?
5. State ***Post's Theorem*** as well as several useful "corollaries" of this result.
6. What is an ***arithmetic predicate***? What does it mean for the quantifiers associated with an arithmetic predicate to be ***alternating***?
7. State an important relationship between arithmetic predicates and Σ_n-predicates and Π_n-predicates.
8. State an important relationship between the sets in the arithmetic hierarchy and arithmetic predicates.

Problems To Be Solved

1. Let
$$\text{SameDomain} = \{n \in \mathbb{N} \mid W_{\ell(n)} = W_{r(n)}\},$$
that is, SameDomain is the set of encodings of pairs of natural numbers x and y such that the partial functions computed by the $\mathcal{S}$-programs encoded by x and by y are defined on the same set of values.
    (a) Prove that SameDomain $\in \Pi_2$.
    (b) Use a reduction, and a result proved in Lecture #16, to prove that SameDomain is 1-complete for Π_2.