Lecture #18: Introduction to Post's Problem
Exercises and Review

Questions for Review

Degrees of Unsolvability — or "Q-Degrees"

1. Suppose that Q is a reducibility. What is a ***Q-degree***?
2. What is a ***Turing degree***?
3. How many Turing degrees exist (that is, what is the "size" or "cardinality" of the set all Turing degrees)?
4. How many Turing degrees include sets in the Arithmetic Hierarchy (that is, what is the "size" or "cardinality" of the set of all Turing degrees that include sets in the Arithmetic Hierarchy)?
5. Name and describe ***two*** Turing degrees that include sets that are computably enumerable.
6. What is ***Post's problem***?
7. Give two equivalent statements of Post's problem.

One-One Equivalence, Recursive Isomorphism, and Myhill's Theorem

8. What is ***recursive permutation***?
9. Suppose $A, B \subseteq \mathbb{N}$. What does it mean for A to be ***recursively isomorphic*** to B, that is, $A \equiv B$?
10. What is ***Myhill's Theorem***? What does imply about the relationship between ***recursive isomorphism*** ("$\equiv$") and ***$\mathbf{1}$-equivalence*** ("$\equiv_1$")?
11. Name and describe an ***infinite collection*** of 1-degrees that include computably enumerable sets.

Cylinders

12. What is the ***cylindrication*** *Cyl*(A) of a set $A \subseteq \mathbb{N}$?

13. What does it mean for a set $A \subseteq \mathbb{N}$ to be a ***cylinder***?

14. Describe ***two other conditions*** that are provably equivalent to the condition "A is a cylinder," for a subset A of $\mathbb{N}$.

15. Suppose that every set $A \subseteq \mathbb{N}$, that is computably enumerable but not computable, is a cylinder. What does this imply about the relationship between 1-degrees, including sets that are computably enumerable but not computable, and m-degrees including such sets?

16. Suppose, instead there exists one computably enumerable set $A \subseteq \mathbb{N}$ such that A is not computable and A is *not* a cylinder. What does *this* imply about the relationship between 1-degrees, including sets that are computably enumerable but not computable, and m degrees including such sets?

Problems To Be Solved

1. Prove that every m-degree S includes a ***maximal*** 1-degree, that is, a 1-degree T such that $A \preceq_1 B$ for all $A \in S$ and $B \in T$. In this particular, this is true when

$$T = \{\textit{Cyl}(A) \mid A \in S\}.$$

2. Prove that if every set A that is computably enumerable, but not computable, is also a cylinder, then every m-degree including sets that are computably enumerable, but not computable, is also a 1-degree.

3. Prove that if there exists a set $A \subseteq \mathbb{N}$ that is computably enumerable, but not computable, and A is also *not* a cylinder, then there exists an m-degree, consisting of sets that are computably enumerable, but not computable, that contains more than one 1-degree.