Lecture #21: Chomsky Hierarchy I — Regular Languages
Lecture Presentation

Establishing the Language of a Right-Regular Grammar

Let $G = (V, \Sigma, \Pi, S_0)$ where $\Sigma = \{\mathtt{a}, \mathtt{b}\}$, $V = \{S_0, S_1, S_2\}$, S_0 is the start variable, and Π includes the following productions:

(i) $S_0 \to \mathtt{a}S_2$	(iv) $S_2 \to \mathtt{a}S_1$	(vi) $S_1 \to \mathtt{a}S_0$
(ii) $S_0 \to \mathtt{b}S_0$	(v) $S_2 \to \mathtt{b}S_2$	(vii) $S_1 \to \mathtt{b}S_1$
(iii) $S_0 \to \lambda$		

It was claimed, in the preparatory reading, that

$$L(G) = \{\omega \in \Sigma^\star \mid \text{the number of copies of “}\mathtt{a}\text{” in } \omega \text{ is divisible by } 3\}.$$

The goal of the first part of this presentation is to sketch a proof of this claim.

Claim 1. *Let $\omega \in \Sigma^\star$.*

(a) If the number of copies of “$\mathtt{a}$” in ω is divisible by 3 then $S_0 \Rightarrow^\star \omega$.

(b) If the number of copies of “$\mathtt{a}$” in ω is congruent to 1 mod 3 then $S_1 \Rightarrow^\star \omega$.

(c) If the number of copies of “$\mathtt{a}$” in ω is congruent to 2 mod 3 then $S_2 \Rightarrow^\star \omega$.

How This Can Be Proved:

Proof of Claim:

Claim 2. *Let* $\omega \in \Sigma^\star$*.*

(a) If $S_0 \Rightarrow^\star \omega$ *then the number of copies of "*$\mathtt{a}$*" in* ω *is divisible by* 3*.*

(b) If $S_1 \Rightarrow^\star \omega$ *then the number of copies of "*$\mathtt{a}$*" in* ω *is congruent to* 1 *mod* 3*.*

(c) If $S_2 \Rightarrow^\star \omega$ *then the number of copies of "*$\mathtt{a}$*" in* ω *is congruent to* 2 *mod* 3*.*

How This Can Be Proved:

Proof of Claim:

Completion of the Proof:

Relating Right-Regular Grammars and Deterministic Finite Automata

A Simplified Kind of Right-Regular Grammar

Lemma #1: Let $G = (V, \Sigma, \Pi, S)$ be a right-regular grammar. Then there exists a right-regular grammar

$$\widehat{G} = (\widehat{V}, \Sigma, \widehat{\Pi}, \widehat{S})$$

which satisfies the following properties.

- Each of the productions in $\widehat{\Pi}$ either has the form $A \to \sigma B$, for $A, B \in \widehat{V}$ and $\sigma \in \Sigma$, or has the form $A \to \lambda$, for $A \in \widehat{V}$. That is, $\widehat{\Pi}$ does not include any productions with the form $A \to \sigma$ for $A \in \widehat{V}$ and $\sigma \in \Sigma$.
- $L(\widehat{G}) = L(G)$.

How To Prove This:

Relating Right-Regular Grammars and Nondeterministic Finite Automata

Lemma #2: Let $\widehat{G} = (\widehat{V}, \Sigma, \widehat{\Pi}, \widehat{S})$ be a right-regular grammar such that each of the productions in $\widehat{\Pi}$ either has the form $A \to \sigma B$, for $A, B \in \widehat{V}$ and $\sigma \in \Sigma$, or has the form $A \to \lambda$, for $A \in \widehat{V}$. Then there exists a ***nondeterministic finite automaton***

$$M = (Q, \Sigma, \delta, q_0, F)$$

such that $L(G) = L(M)$.

How To Prove This:

Relating Deterministic Finite Automata and Right-Regular Grammars

Lemma #3: Let $M = (Q, \Sigma, \delta, q_0, F)$ be a ***deterministic finite automaton***. Then there exist a ***right-regular grammar*** $G = (V, \Sigma, \Pi, S)$ such that $L(M) = L(G)$.

How To Prove This:

Completing a Proof of Claim #1:

Claim #1: Let $L \subseteq \Sigma^\star$ (for an alphabet $\Sigma^\star$). Then the following properties are equivalent:

- L is the language of a right-regular grammar G — so that L is a ***regular language***, as defined in these notes.
- L is the language of a deterministic finite automaton — so that L is a "regular language", as these were defined in CPSC 351 (or CPSC 313).

Completion of Proof:

Decidable Problems and Computable Functions

Testing Membership and Parsing

Other Problems