Lecture #23: Chomsky Hierarchy III — Context-Sensitive Languages

Exercises and Review

Questions for Review

1. What is a ***context-sensitive grammar***? What is a ***context-sensitive language***?
2. Describe, as precisely as you can, how the set of ***context-sensitive languages*** and the set of ***context-free languages*** are related.
3. Describe, as precisely as you can, how the set of ***context-sensitive languages*** and the set of ***decidable*** languages are related.
4. What is a ***noncontracting grammar***? What is an ***essentially noncontracting grammar***?
5. Describe, as precisely as you can, the relationships between the following sets of languages:
   - the set of all languages of ***noncontracting grammars***
   - the set of all languages of ***essentially noncontracting grammars***
   - the set of all ***context-sensitive languages that do not contain the empty string***
   - the set of all ***context-sensitive languages***
6. What is ***Kuroda normal form***? How are the languages of grammars, that are in Kuroda normal form, related to the sets of languages that have already been considered?
7. What is a ***linear bounded automaton***? How is it related to a ***nondeterministic Turing machine*** — and how are the languages of linear bounded automata related to the sets of languages that have already been considered?
8. Define the complexity class **NSPACE**(n). How is this complexity class related to the sets of languages that have already been considered? How is this fact useful?
9. Describe (at least) three or four ***closure properties*** for the set of context-sensitive languages.

10. Describe a ***solvable*** decision problem that involves context-sensitive grammars and their languages, as well as an ***unsolvable*** decision problem that involves these grammars and languages.

Problems To Be Solved

1. Working from the material in the lecture notes, complete a proof that every language, that is the language of an essentially noncontracting grammar, is context-sensitive.

   Note: A process that was used to complete a proof, in the lecture presentation, might be useful if you try to solve this problem.

2. Prove that the language of every noncontracting grammar is also the language of a grammar in Kuroda normal form.

3. Consider a linear bounded automaton $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ and a string $\omega \in \Sigma^\star$ with length n.

   (a) Give an upper bound — that will almost certainly depend on $|Q|$ and $|\Gamma|$, as well as n — for the number of ***configurations*** that M can be in, during an execution on the input string ω.

   (b) Notice that this gives a way to decide that a computation on input ω that already includes a certain number of steps is "looping", in some sense, and is never going to terminate.

   Use this to describe a — provably correct — way to traverse the ***computation tree*** for M and ω, in a way that allows you to decide whether $\omega \in L(M)$.