Cellular Automata

and beyond ...

The World of Simple Programs

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Cellular Automata

Lindenmayer Systems

Random Boolean Networks

Classifier Systems
Cellular Automata

Global Effects from Local Rules
Cellular Automata: Local Rules — Global Effects

Demos
Cellular Automata

- The CA space is a lattice of cells (usually 1D, 2D, 3D) with a particular geometry.
- Each cell contains a variable from a limited range of values (e.g., 0 and 1).
- All cells update synchronously.
- All cells use the same updating rule (in uniform CA), depending only on local relations.
- Time advances in discrete steps.
One-dimensional Finite CA Architecture

- Neighbourhood size: $K = 5$
  - local connections per cell
- Synchronous update in discrete time steps

Time Evolution of Cell $i$ with K-Neighbourhood

$$C_i^{(t+1)} = f(C_i^{(t)}-\left[\frac{K}{2}\right], \ldots, C_{i-1}^{(t)}, C_i^{(t)}, C_{i+1}^{(t)}, \ldots, C_{i+\left[\frac{K}{2}\right]}^{(t)})$$

With periodic boundary conditions:

$$x < 1 : C_x = C_{N+x}$$

$$x > N : C_x = C_x - N$$
Value Range and Update Rules

- For $V$ different states (= values) per cell there are $V^K$ permutations of values in a neighbourhood of size $K$.

- The update function $f$ can be implemented as a lookup table with $V^K$ entries, giving $V^{V^K}$ possible rules.

<table>
<thead>
<tr>
<th>$v$</th>
<th>$K$</th>
<th>$V^K$</th>
<th>$V^{V^K}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>8</td>
<td>256</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>32</td>
<td>$4.3 \times 10^9$</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>128</td>
<td>$3.4 \times 10^{38}$</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>512</td>
<td>$1.3 \times 10^{154}$</td>
</tr>
</tbody>
</table>
Cellular Automata: Local Rules — Global Effects

Demos
History of Cellular Automata

• Alternative names:
  – Tessellation automata
  – Cellular spaces
  – Iterative automata
  – Homogeneous structures
  – Universal spaces

• John von Neumann (1947)
  – Tries to develop abstract model of self-reproduction in biology (from investigations in cybernetics; Norbert Wiener)

• J. von Neumann & Stanislaw Ulam (1951)
  – 2D self-reproducing cellular automaton
  – 29 states per cell
  – Complicated rules
  – 200,000 cell configuration
  – (Details filled in by Arthur Burks in 1960s.)
• Threads emerging from J. von Neumann’s work:
  – Self-reproducing automata (spacecraft!)
  – Mathematical studies of the essence of
    • Self-reproduction and
    • Universal computation.

• CAs as Parallel Computers (end of 1950s / 1960s)
  – Theorems about CAs (analogies to Turing machines) and their formal computational capabilities
  – Connecting CAs to mathematical discussions of dynamical systems (e.g., fluid dynamics, gases, multi-particle systems)

• 1D and 2D CAs used in electronic devices (1950s)
  – Digital image processing (with so-called cellular logic systems)
  – Optical character recognition
  – Microscopic particle counting
  – Noise removal
History of Cellular Automata (3)

• Stansilaw Ulam at Los Alamos Laboratories
  – 2D cellular automata to produce recursively defined geometrical objects (evolution from a single black cell)
  – Explorations of simple growth rules

• Specific types of CAs (1950s/60s)
  – 1D: optimization of circuits for arithmetic and other operations
  – 2D:
    • Neural networks with neuron cells arranged on a grid
    • Active media: reaction-diffusion processes

• John Horton Conway (1970s)
  – Game of Life (on a 2D grid)
  – Popularized by Martin Gardner: *Scientific American*
Stephen Wolfram's World of CAs

1976-1982: Quarks and beyond...
Stephen Wolfram’s World of CAs

1981 - 1985: Discoveries about cellular automata...
Stephen Wolfram's World of CAs
Stephen Wolfram’s World of CAs
Example Update Rule

- $V = 2$, $K = 3$

- The rule table for rule 30:

<table>
<thead>
<tr>
<th>111</th>
<th>110</th>
<th>101</th>
<th>100</th>
<th>011</th>
<th>010</th>
<th>001</th>
<th>000</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

$128 + 64 + 32 + 16 + 8 + 4 + 2 + 1 = 30$

See examples ...
CA Demos

- *Evolvica CA Notebooks*
Four Wolfram Classes of CA

- **Class 1:**
  A fixed, homogeneous, state is eventually reached (e.g., rules 0, 8, 128, 136, 160, 168).
Four Wolfram Classes of CA

- **Class 2:**
  A pattern consisting of separated periodic regions is produced (e.g., rules 4, 37, 56, 73).
Four Wolfram Classes of CA

- **Class 3:**
  A chaotic, aperiodic, pattern is produced (e.g., rules 18, 45, 105, 126).
Four Wolfram Classes of CA

- **Class 4:**
  Complex, localized structures are generated (e.g., rules 30, 110).
Class 4: Rule 30
Class 4: Rule 110
Further Classifications of CA Evolution

- Wolfram classifies CAs according to the patterns they evolve:
  - 1. Pattern disappears with time.
  - 2. Pattern evolves to a fixed finite size.
  - 3. Pattern grows indefinitely at a fixed speed.
  - 4. Pattern grows and contracts irregularly.

- Qualitative Classes
  - 1. Spatially homogeneous state
  - 2. Sequence of simple stable or periodic structures
  - 3. Chaotic aperiodic behaviour
  - 4. Complicated localized structures, some propagating
Further Classifications of CA Evolution (2)

- Classes from an Information Propagation Perspective
  - 1. No change in final state
  - 2. Changes only in a finite region
  - 3. Changes over an ever-increasing region
  - 4. Irregular changes

- Degrees of Predictability for the Outcome of the CA Evolution
  - 1. Entirely predictable, independent of initial state
  - 2. Local behavior predictable from local initial state
  - 3. Behavior depends on an ever-increasing initial region
  - 4. Behavior effectively unpredictable
The “Game of Life”: a 2D Cellular Automaton

John Horton Conway’s Game of Life
• Wuensche, A. Discrete Dynamics Lab: http://www.santafe.edu/~wuensch/ddlab.html