CPSC/PMAT 669

Classical Crytosystems

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Topic 1

Outline

Motivation

Cryptography (from the Greek) — 'hidden writing'

What would you like to see in a secure electronic assignment submission system? Want submission:

- confidential so no one can steal it (confidentiality)
- protected so no one can alter it (data integrity)
- authentic so no one can impersonate creator (entity authentication)
- safe from intrusion on disk (access control)
- safe from denial by instructor or TA (non-repudiation)

This course will work toward solutions for ensuring all of these. Examples of complete systems at end of the course.

Resources

Course web page (link from instructor's home page):

- Course info, assignments, handouts, course schedule, useful links
- D2L: grade reporting *only*

Resources:

- recommended texts: Menezes, van Oorschot, and Vanstone, Stinson (3rd ed.), Katz & Lindell, Paar & Pelzl
- other sources on web (see course web pages, in particular the "links" page)

Evaluation

40%: 3 assignments

- mostly testing theoretical cocepts, some more applied questions
- all work must be done *individually*

60%: research project (samples, requirements available online)

- proposal (10%)
- paper (40%)
- in-class presentation (10%)

Basic Terminology

Historically, cryptography is the art of sending messages in secret, or disguised form.

Definition 1 (encrypt, encipher)

To render a message unintelligible except to the intended recipient.

Definition 2 (decrypt, decipher)

To transform an encrypted message back into its unencrypted form.

More Terminology

Definition 3 (plaintext, cleartext, "in the clear")

The message or data to be encrypted.

Definition 4 (ciphertext, cryptogram)

The message after encryption.

Definition 5 (cipher, cryptosystem)

A particular method of encryption, capable of handling arbitrary messages

An Old Example

Example 6 (Caesar Cipher)

Substitute each plaintext letter with the third subsequent letter of the alphabet, wrapping from Z to A; *i.e.* $A \to D$, $B \to E$, $\cdots$, $Z \to C$. :

Plaintext: `I came, I saw, I conquered.`

Ciphertext: `L FDPH, L VDZ, L FRQTXHUHG.`

Example of a class of ciphers knows as *shift ciphers*:

- shift every letter by another letter a fixed position down in the alphabet (with "wrap-around") at "Z").

2000 years old: According to Suetonius ("Lives of the Caesars"), Julius Caesar used this cipher during his campaign in Gaul (modern day France) to send encrypted dispatches back to Rome.

Who Uses Cryptography?

Historic users:

- governments (military, diplomatic service)
- a illicit private uses (secret love letters, conspiracies)

Modern users (since invention of computers):

- everyone! (everyone using a computer, smart phone, credit card, BluRay player, . . .)

Cryptography is ubiquitous! Examples:

- e-commerce, online banking, online purchases, online auctioning (eBay), logging into a computer, using a banking machine, and many more.

Modern cryptography does MUCH more than just hiding messages.

Recreational Reading

For cryptography in history and literature, Simon Singh's *The Code Book* (Doubleday 1999) is highly recommended. See also Singh's website `www.simonsingh.net`.

The most comprehensive source on cryptography in military history is David Kahn's *The Code Breakers* (1967).

Information Security

Definition 7 (information security)

Measures to protect information and information systems from unauthorized access, use, disclosure, disruption, modification or destruction.

Cryptography provides *some* such measures

- important part of complete security systems
- does *not* do it all!

Security Objectives

Services provided by modern cryptography:

- Data confidentiality (data only readable to legitimate parties)
- Data integrity (data has not been modified)
- Non-repudiation (protection against denial by one of the parties in a communication)
- Authentication (communicating entity is the one claimed)
- Access Control

Security Mechanisms

Encryption is just one of many *security mechanisms* that achieve one or more of the above security objective.

Cryptographic security mechanisms discussed in this course include:

- Encryption systems — for confidentiality and limited data integrity
- Digital signatures — for data integrity and non-repudiation
- Hash functions, Message Authentication Codes (MACs) — for data integrity and authentication

Cryptography provides many security mechanisms, but not all

- Necessary, but not sufficient for information security (more later!)
- See Anderson "Why cryptosystems fail" (see "external links").

Security Attacks

Security mechanisms are designed to detect, prevent, or recover from a *security attack*, *i.e.* an action that compromises the security of information owned by an organization.

We distinguish between

- *passive* attacks – listening, eavesdropping on information
- *active* attacks – modifying information (for impersonation, replaying messages, changing contents, or denial of service)

Successful cryptographic protocols typically combine several mechanisms to guard against as many different attacks as possible (especially active ones).

Modern Terminology

Definition 8

Cryptography – the study of mathematical techniques for providing information security services

Cryptanalysis – the study of mathematical techniques for attempting to defeat cryptographic security mechanisms

Cryptology – combined fields of cryptography and cryptanalysis

Cryptographic primitive – tool that represents a cryptographic security mechanism

Cryptographic protocol – an algorithm (sequence of steps) to be undertaken by two or more entities to achieve a specific security objective

Will cover primitives/protocols for all security mechanisms listed above.

Great reference: *Handbook of Applied Cryptography* (see "external links")

Terminology

Definition 9

Message space $\mathcal{M}$ – set of all possible plaintext messages

Ciphertext space $\mathcal{C}$ – set of all possible encrypted messages

Key space $\mathcal{K}$ – the finite set of possible keys

Encryption transformation – a left invertible map $E_K : \mathcal{M} \to \mathcal{C}$, indexed by some key $k \in \mathcal{K}$

Decryption transformations – the left inverse map D_K of E_K, so $D_K(E_K(M)) = M$ for all plaintexts $M \in \mathcal{M}$.

Note: $D_K(E_K(M)) = M$ implies that $D_K \circ E_K = I$ is the *identity transformation* on $\mathcal{M}$.

Note: The fact that E_K is left-invertible is equivalent to E_K is an *injective* (*i.e.* one-to-one) map.

The Idea of Encryption and Decryption

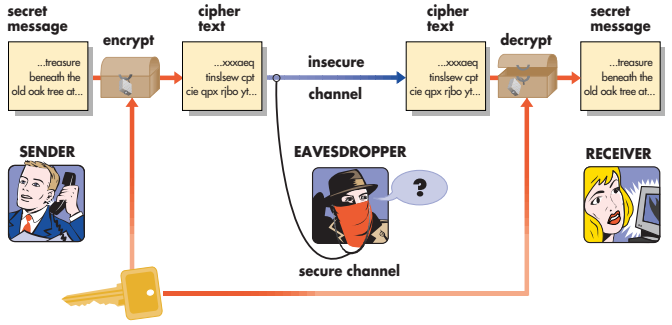
Gilles Brassard, a professor at the Université de Montréal and the inventor of quantum cryptography, created the protagonists *Alice* and *Bob*.

- Since then, many more characters have joined the crypto game; most notably *Eve*.

Idea:

- A transmitter (Bob) generates a plaintext $M \in \mathcal{M}$, to be communicated to a legitimate receiver (Alice) over an insecure channel.
- To prevent an eavesdropper (Eve) from learning the contents of M, Bob chooses a key $K \in \mathcal{K}$ and encrypts M with E_K to produce the ciphertext $C = E_K(M)$.
- C is sent along the insecure channel. When Alice obtains C, she deciphers it by applying D_K to C to obtain $M = D_K(C)$.

Conventional Cryptosystem



Issues

Encryption functions are our first example of a cryptographic primitive

- could easily formalize the above description to create a cryptographic protocol.

Note that Bob must somehow communicate the secret key to Alice without Eve obtaining it, *i.e.* over a secure channel (more on that later).

The assumption is that the workings of E_K and D_K are not secret, but K is secret. So only Alice and Bob can encrypt and decrypt, but no one else can.

Example: Shift Cipher

Description:

- $\mathcal{M} = \mathcal{C} = \{A, B, \ldots, Z\}$.
- Keys represent shifts by a position between 0 and 25.
- Encryption is a forward circular shift of a plaintext letter by K
- Decryption is the corresponding backward circular shift of a ciphertext letter by K.

Example, cont.

More formally, first assign each letter a numerical equivalent as follows.

0	1	2	3	...	25
a	b	c	d	...	z

Message, cipher, and key space: $\mathcal{M} = \mathcal{C} = \mathcal{K} = \mathbb{Z}_{26}$ (integers modulo 26).

Encryption: $E_K(M) \equiv M + K \pmod{26}$ (remainder between 0 and 25).

Decryption: $D_K(C) \equiv C - K \pmod{26}$ (remainder between 0 and 25).

For the Caesar cipher, $K = 3$.

Issues with the Shift Cipher

Main problem: very small key space ($|\mathcal{K}| = 26$)

- Easily falls to a "brute force attack" by simply trying each key in turn. (assumes that you know that a shift cipher is used)

Note: How small is "small?"

- With modern technology, one tenth of a billion billion billion = $10^{17} \approx 2^{56}$ is small (DES has $|\mathcal{K}| = 2^{56}$).
- Clearly, $26 < 2^{56}$. (2^{80} questionable!)

Symmetric Cryptosystems

We are now in a position to formally define a cryptosystem.

Definition 10 (Symmetric Cryptosystem)

A single-parameter family $\{E_K\}_{K \in \mathcal{K}}$ of injective transformations

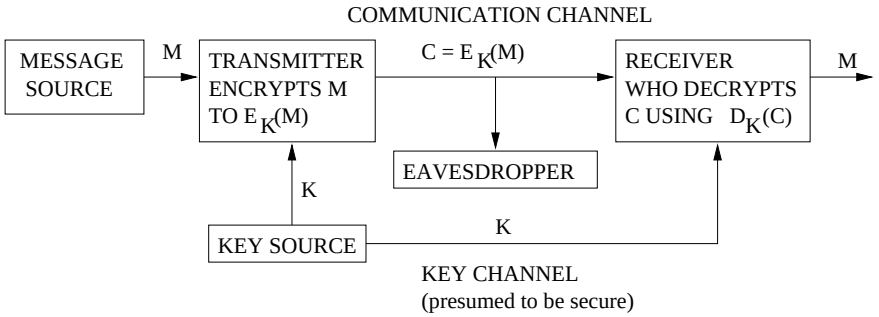
$$E_K : \mathcal{M} \to \mathcal{C}$$
$$M \mapsto E_K(M) = C \quad (M \in \mathcal{M}, C \in \mathcal{C}),$$

where E_k acts on a *message-space* $\mathcal{M}$ and injects it into a *cipher-space* $\mathcal{C}$.

- The parameter or key K is selected from the *key space* $\mathcal{K}$.
- For any $K \in \mathcal{K}$, the left inverse of E_K is denoted D_K.

Schematic of a Symmetric Cryptosystem

AKA *conventional* or *private key* cryptosystems.



Key Channel

In order for the encryption to be secure, *key channels* must be absolutely secure, as must the channel from the source to the transmitter.

In the real world, this usually means expensive.

For example, the keys to the Moscow-Washington hotline are transmitted by means of highly paid couriers, who fly there and back every week.

But If We Already Have a Secure Channel...?

It would be nice to dispense with the key channel. Why bother encrypting when we have a secure channel already?

- *Time-shifting, convenience* – you have access to a secure channel now, but would like to use it later, when the channel may not be available.
- *Speed, bandwidth* – the secure channel may be slow or of a limited bit rate.
- *Cost* – the secure channel may be expensive; *e.g.* hand-delivered by courier.
- *Feasibility* – the secure channel may be impractical; *e.g.* Alice and Bob meet in person before securely communicating.

Goals of an Attacker

We can now refine our notions of attacks on cryptosystems

Goals of an attacker:

- Deduce the key or portions thereof
- Deduce one or more plaintexts or portions thereof
- Modify a message
- Replay a message
- Impersonate (i.e., masquerade as) another entity

The first two are passive attacks, the last three active attacks.

Types of Attacks on Cryptosystems

Depends on what adversary has available and what he/she can do.

- *Ciphertext Only Attack* (COA) – adversary has only ciphertext, but no plaintext.
- *Known Plaintext Attack* (KPA) – adversary has some plaintext and corresponding ciphertext.
- *Chosen Plaintext Attack* (CPA) – adversary has some plaintext of his choosing and the corresponding ciphertext.

Types of Attacks, cont.

- *Adaptive* CPA – adversary's choice of plaintext may depend on ciphertexts received from previous requests.
- *Chosen Ciphertext Attack* (CCA) – adversary chooses some ciphertext and is then given the corresponding plaintext. He is not allowed to chose the ciphertext he wishes to decrypt.
- *Adaptive* CCA (CCA2) – adversary's choice of ciphertext may depend on plaintexts received from previous requests

COA and the known text attacks are passive; the chosen text attacks and their adaptive versions are active.

More on Attacks

Note: A good/secure cryptosystem should be be secure against adaptive CCA's (as strong as possible)

Some attacks that cryptography cannot protect against:

- *Side Channel Attacks* – adversary exploits some physical aspect of the cryptosystem implementation to extract the key (power/timing/radiation analysis)
- *Clandestine Attacks* (AKA Rubber Hose Cryptography) – adversary bribes, blackmails, threatens, steals, or beats the key out of the recipient

Notions of Security

Definition 11 (Kerckhoff's Principle)

The security of a cryptosystem should depend entirely upon knowledge of the key, not of the method.

- From "La Cryptographie Militaire" (1883), one of the first scientific treatments of cryptography.
- This implies in particular that a cipher should be completely published and still be secure (against its own designer and everyone else).

So what constitutes a *secure* cryptosystem? We saw that a good system should be secure against adaptive CCA's. What does "secure" mean? There are different notions of security.

Measures of Security

Listed from strongest to weakest:

- *Unconditional* Security – can an adversary with unlimited computing power defeat the system?
- *Provable* Security – breaking the system can be reduced (mathematically) to another, supposedly difficult problem; *e.g.* integer factorization.
- *Computational* Security – does the perceived amount of computing power necessary to break the system (using the best known method) exceed (by a comfortable margin) the available computing power of the attacker?
- *Ad-hoc* Security – security is "proved" via a series of convincing arguments that every successful attack is impractical.

Remarks

Computational security often used in conjunction with provable security

- Eg. a typical security claim might read something like "a cryptosystem is provably secure against an adaptive CCA assuming integer factorization is hard"

Provable security does *not* mean that a cryptosystem is *proved* secure!

- Proofs typically only reduce to another problem (which could eventually be solved)
- Proofs assume specific adversarial capabilities and attacks (eg. adaptive CCA)

Classical Ciphers

Classical ciphers usually belong to one of the following two types: substitution or transposition ciphers.

Definition 12 (Substitution cipher)

A cipher for which encryption replaces each plaintext symbol by some ciphertext symbol without changing the order of the plaintext symbols.

Definition 13 (Transposition cipher)

A cipher in which the ciphertext is a rearrangement (*i.e.* permutation) of the plaintext symbols.

Modern Usage

Individually, substitution ciphers and transposition ciphers are generally insecure.

However, when alternating them repeatedly,

$$M \longrightarrow \boxed{\text{T}} \longrightarrow \boxed{\text{S}} \longrightarrow \boxed{\text{T}} \longrightarrow \boxed{\text{S}} \longrightarrow \cdots \longrightarrow \boxed{\text{T}} \longrightarrow \boxed{\text{S}} \longrightarrow C \ ,$$

they become very secure.

This is how modern symmetric cryptosystems are designed — more later!

Monoalphabetic Substitution Ciphers

Definition 14 (Monoalphabetic Substitution cipher)

A substitution cipher that uses a single ciphertext alphabet.

Examples:

- shift cipher
- assign cipher character (english characters or otherwise) to each plaintext character
- substitute by pairs (digraphs). Eg. Playfair cipher

Fiction literature is full of examples of monoalphabetic substitution ciphers:

- Edgar Allan Poe's *The Gold Bug*
- Sir Arthur Conan Doyle's *The Adventure of the Dancing Men*
- Even the bible contains examples, derived from a cipher called *atbash*

Security of Monoalphabetic Substitution Ciphers

Monoalphabetic substitution ciphers are in general completely insecure:

1. Highly vulnerable to KPA's. Each portion of corresponding plaintext and ciphertext reveals some of the cipher.
   - Eg. For shift ciphers, one corresponding plaintext-ciphertext pair actually reveals the key!
2. Each plaintext letter is encrypted to the same ciphertext letter.
   - Thus, frequent ciphertext letters correspond to common plaintext letters (*e.g.* "e" in English).
   - Also pairs of identical ciphertext letters correspond to such paintext letter pairs (*e.g.* "XX" corresponds to "oo")

Security, cont.

3. Redundancy in any language generally yields the key, given a sufficient amount of ciphertext (COA).
   - frequency distribution of the plaintext alphabet (letters, pairs of letters, triples of letters etc.) in a given language can be established statistically and compared with the ciphertext (see frequency and digraph handouts).
   - The method is called the *phi-statistic*. The concept of redundancy can be mathematically formalized.

Of course this all assumes "normal" text.

- Pathological example: *Gadsby*, by Ernest Vincent Wright. This 50,000 word novel is written entirely without using the letter *E*.

Codes

Definition 15 (Code)

A technique by which words or letter combinations are replaced by a set of predetermined codewords.

Codes are essentially monoalphabetic substitution ciphers with very large plaintext alphabets.

Historical examples:

- Mary Queen of Scotts conspiring to overthrow Queen Elizabeth I and gain the English throne
- Famous 1917 WW I Zimmerman telegram
- Navajo Code talkers in WW II

Polyalphabetical Substitution Ciphers

Definition 16 (polyalphabetic substitution cipher)

A substitution cipher in which several cipher alphabets are used in the replacement of the plaintext characters.

Example 17

The Vigenère Cipher: Originally described by Giovan Batista Belaso (1553) in *La cifra del. Sig. Giovan Batista Belaso*. Rediscovered many times. To the French, it became known as *le chiffre indéchiffrable* ('the unbreakable cipher'). It is basically a collection of shift ciphers, each corresponding to a letter in a key word.

Other Polyalphabetic Substitution Ciphers

Beauford cipher – slight variant of Vigenère

Mixed Vigenère – collection of shifted monoalphabetic substitution ciphers, each corresponding to a key word.

- Harder to cryptanalyze than ordinary Vigenère – need to use a technique called *symmetry of position* to find out the column permutation – but still insecure.

Coherent Running Key cipher – like a Vigenère cipher but with a "running" *i.e.* very long) key, usually taken from a readily available text.

- Still falls to frequency analysis due to language redundancy. However, it has been proven that multiple encryption using four different running keys produces a statistically secure cipher.

Transposition Ciphers

Recall that a transposition cipher is a rearrangement (permutation) of the plaintext letters.

Definition 18 (Route cipher)

A transposition cipher where the plaintext is arranged in some geometric figure and the ciphertext is obtained by rearranging the plaintext according to some route through the figure.

Definition 19 (Columnar Transposition)

The message is arranged horizontally in a rectangle. The key is used to generate a permutation of the columns. The ciphertext is read vertically from the permuted columns.

Cryptanalysis of Columnar Transposition

Vulnerable to a COA:

- Guess the dimensions of the rectangle
- Determine the order of the columns via frequency counts (which will be the same as for English text). Place columns adjacent to each other if they produce common latter pairs (*e.g.* QX is extremely unlikely, but EN is highly likely).