1. Construct a 3-d tree using the following dimensions: age (int), years with the company (int), salary (real) for the following database: John(60, 24, 64,000); Scott(25, 2, 50,000); Charlie(38, 18, 54000); David(55, 29, 68,400); Ellen(27, 7, 55000); Frank(57, 17, 115000); Grant (66, 22, 40000).

Answer- Here we assume that Blocks can hold maximum 2 records. Notice that after we split by years, each branch has maximum two records and we can stop here. However, since the question asks for a 3-d tree we go one step further. The following shows the tree:

$$
\begin{gathered}
\text { Scott( } 25,2,50,000) \\
\text { Ellen }(27,7,55000) \\
\text { Charlie }(38,18,54000) \\
\text { David }(55,29,68,400) \\
\text { Frank }(57,17,115000) \\
\text { John }(60,24,64,000) \\
\text { Grant }(66,22,40000)
\end{gathered}
$$


2. Construct a 2-d tree using age and salary values for Question 1.

3. Represent graphically the 2-d tree from Q2 (geometric interpretation).

4. The principal of a high school decides to reorganize students in classes according to the grades they have obtained in the past year on the following subjects: Math, English, French, History, Geography, Music, Sport. The grades are between 1 and 5 . Students will be divided into 7 classes, one for each subject, with their grades for a specific subject falling into one of the intervals: Math [3.5,5]; English [2.2, 3.7], French [1.7, 2.5]; History [1.0, 2.0]; Geography [3.7, 4.2]; Music [4.2.5]; Sport [2.2.2.7]. Assuming that each student is assigned to only one class according to the above criteria, construct an interval tree of the seven classes.

Answer- We start by taking the entire range of all the intervals [1,5] and dividing it in half at " 2.5 ". This gives three sets of intervals, those completely to the left of "3" (History, French, Sport), those completely to the right of 3 (Geography, Math, and Music), and those overlapping 3 (English). The intervals in the left and right are recursively divided in the same manner until there are no intervals left.

The intervals that overlap the center point are stored in a separate data structure linked to the node in the interval tree.

The result is a binary tree with each node storing:

* A center point
* A pointer to another node containing all intervals completely to the left of the center point
* A pointer to another node containing all intervals completely to the right of the center point
* All intervals overlapping the center


5. Using the interval tree from Q6, search for classes where all students are very good at on subject (marks between [2.5,3.5]).

Answer- Starting from the root, the split point is 3 which falls within our interval, therefore English is included in the result. Moreover, all the ranges to the left and to the right must be checked.

Going to the left, the next split point is 2 . Since 2 is less than our start point the intervals to its left are not of interest and therefore its left child won't be explored. However the areas to its right might have either their start point or endpoint in the required range. This is the case for Sport and thus it is included in the result. The link list at node 2 must also be checked: History has both of its startpoint and endpoint out of the range but French has its endpoint inside the range. Therefore, French is also included in the result.

In the right subtree, the next split point is 4 . Since 4 is greater than our endpoint the intervals to its right are not of interest and therefore its right child won't be explored. The link list at node 4 must also be checked: Math has its start point in the range but geography's start point is outside of our range. Therefore, Math is also included in the result.

The final answer would be: English, Sport, French, and Math.
6. Construct a B-tree of order 3 to insert the following names: Calgary, Vancouver, Toronto, Saskatoon, Lethbridge, Regina, Vernon, Edmonton, Winnipeg.

Answer- The tree is of order 3 (maximum number of children from each node) and each node can hold a maximum of 2 keys.

The leaves must have at least $3 / 2-1=1$ key. The interior nodes must have at least $3 / 2=2$ subtrees. The root must have at least 2 subtrees.

The keys are always kept in ascending order within a node.
Insert Calgary:


Insert Vancouver:


Insert Toronto:


Insert Saskatoon:


Insert Lethbridge:


Insert Regina:


Insert Vernon:


Insert Edmonton:


Insert Winnipeg:

7. Using the B-tree generated at Q8, delete the followings in this order: Toronto, Winnipeg, Edmonton. Show how the B-tree changes every time.

Answer- Deleting Toronto: We begin by finding the immediate successor, which would be Vernon, and move the Vernon up to replace the Toronto.


Now we have to find the immediate successor to Vernon to bring up. This successor would be Winnipeg.


Now in the leaf nodes we have an empty block and we need to merge it with the leaf on the left.


After merging we will have an empty interior node. This node needs to be merged with the node on the left and consequently the tree shrinks in height by one.


Deleting Winnipeg:


Deleting Edmonton:


9 | P a g e
8. Draw a B+tree for question 6 .

Answer- Insert Calgary:


Insert Vancouver:


Insert Toronto:


Insert Saskatoon:


Insert Lethbridge:


Insert Regina:


Insert Vernon:


Insert Edmonton:


Insert Winnipeg:

9. Draw an ISAM tree for question 6 .

Answer- An ISAM index is static. Insertion only affects the leaf nodes. An ISAM file can be created as follows:

10. Insert the following values in the hash table of size 11: $15,17,4,8,20,28,26,5$. Hash function $\mathrm{h}(\mathrm{key})=$ key mod 11; increment function i(key) $=[$ Quotient(key/11) +1] mod 11.

| Key | Hash value | Incremented | Incremented | Incremented |
| :--- | :--- | :--- | :--- | :--- |
| 15 | 4 |  |  |  |
| 17 | 6 |  |  |  |
| 4 | 4 | 5 |  |  |
| 8 | 8 |  |  |  |
| 20 | 9 |  | $9+3=1$ |  |
| 28 | 6 | $6+3=9$ |  |  |
| 26 | 4 | $5+3=7$ | $6+1=7$ | $7+1=8$ |
| 5 | 5 | $5+1=6$ |  |  |

11. Using extendible hashing, insert the following values: $15,17,4,11,21,28,25,5 . H($ key $)=$ key mod $10+$ Quotient (key/10). A bucket can contain no more than 2 records. The index is constructed using the binary representation of the hash value. Show how the buckets split and the index increases.

| Key | Hash value | Hash value in Binary |
| :--- | :--- | :--- |
| 15 | $5+1=6$ | 0110 |
| 17 | $7+1=8$ | 1000 |
| 4 | $4+0=4$ | 0100 |
| 11 | $1+1=2$ | 0010 |
| 21 | $1+2=3$ | 0011 |
| 28 | $8+2=10$ | 1010 |
| 25 | $5+2=7$ | 0111 |
| 5 | $5+0=5$ | 0101 |



12. What is the minimum depth for a B-tree of order 5 that stores 100 values?

Answer-
$4\left(1+5+25+\cdots+5^{\wedge}(x-1)\right)>100$
$\left(1+5+25+\cdots+5^{\wedge}(x-1)\right)>25$
$x-1=2=>x=3$
13. What is the maximum number of values to be stored in a B-tree of order 3 , depth 4.

Answer- The levels would hold 1, 3, 9 and 27 nodes, respectively, for a total of 40 nodes.

Each node would hold 2 data values, so there would be 80 of them.
14. In a grid file using the area defined by $(0,7) \times(0,7)$ the following points are inserted in this order: $A(3,1), B(2,3), C(6,6), D(7,2), E(2,5), F(5,4)$. In the directory, each cell can store up to 2 points. Show how the directory and the cells from the grid file split.
After inserting $A$ and $B$ :


Inserting C:


Inserting D:


Inserting E:


Inserting F:


