Approaching Mathematical Proofs

This is what you do on your own.

This is what you submit on your assignment.

PRIVATE SPACE

$s_n = \frac{1}{n+1}$

Want to show $\lim_{n\to\infty} s_n = 0$

So we need to find N so that beyond N (meaning $n > N$),

$$|s_n - L| < \varepsilon$$

Where $L = 0$:

$$\left|\frac{1}{n+1} - 0\right| < \varepsilon.$$

We are looking for a bounding N.

Work with inequality:

$$\left|\frac{1}{N+1} - 0\right| < \varepsilon.$$

$$\left|\frac{1}{N+1} - 0\right| < \varepsilon$$

$$\left|\frac{1}{N+1}\right| < \varepsilon \quad N \geq 0$$

$$\frac{1}{N+1} < \varepsilon \quad N+1 \neq 0$$

$$N+1 > \frac{1}{\varepsilon}$$

$$N > \frac{1}{\varepsilon} - 1$$

Try $N(\varepsilon) = \frac{1}{\varepsilon} - 1.$

PUBLIC SPACE

Claim. Let $s_n = \frac{1}{n+1}$, $n \geq 0$. Then $\lim_{n\to\infty} s_n = 0$.

Proof. Let $\varepsilon > 0$

$N(\varepsilon) = \frac{1}{\varepsilon} - 1.$ ✱

Then $n > N(\varepsilon)$ implies:

$$n > N = \frac{1}{\varepsilon} - 1$$

$$n+1 > \frac{1}{\varepsilon}$$

Note $n \geq 0$, so $n+1 > 0$.

$$\frac{1}{n+1} < \varepsilon$$

$$\left|\frac{1}{n+1} - 0\right| < \varepsilon.$$

Hence for all $\varepsilon > 0$, we have $N(\varepsilon) \in \mathbb{R}$ (see ✱) such that $n > N$ implies $|s_n - 0| < \varepsilon$.

By defn of limit, we have $\lim_{n\to\infty} s_n = 0$. □

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