

Elementary Set Theory

Peeking into Computer Science



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- Mandatory: Chapter 2 – Sections 2.3 and 2.4

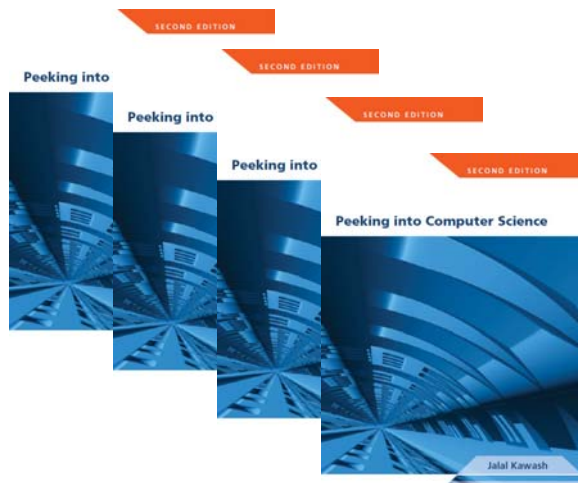


Reading Assignment

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Sets and Set Operations

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At the end of this section, the you will be able to:

1. Understand the two basic properties of sets
2. Understand the (one) difference between sets and multisets
3. Define a set and use two types of set representations
4. Perform all 4 operations on sets
5. Use Venn diagrams to depict sets and operations on them
6. Define tuples and differentiate them from sets
7. Perform multiplication on sets (Cartesian products)

Objectives

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- A **set** is a collection of objects;
 - we call these objects the *elements* of the set.
- Examples:
 - Students in this class form (ie, are elements of) a set
 - The set of all positive integers
 - The set of all of your shirts
- Sometimes we call an element of a set a *member* of that set, instead.

Sets

- Small sets can be represented by listing its members
- $A = \{\text{paper, scissors, rock}\}$
 - All possible choices in the game
- $B = \{\text{pawn, rook, knight, bishop, queen, king}\}$
 - All chess pieces
- $C = \{1, 2, 3, 4, 5\}$
 - All positive integers less than or equal to 5

Representing Small Sets

- May not be possible to list all members
- Some sets are infinite

- $A = \{x \mid x \text{ is a current student at UofC}\}$
 - All current students at UofC

- $B = \{x \mid x \text{ is an even number}\}$
 - Infinite set



Representing Big Sets

- An empty set contains no elements.
- Notation:
 $A = \{\}$
 $A = \phi$



The Empty Set

- Duplicates are not allowed (or do not count)
 - {paper, scissors, rock} is the same as {paper, scissors, rock, rock}
 - We should not repeat elements
- Order of members is irrelevant
 - {paper, scissors, rock}
 - {scissors, rock, paper}
 - {rock, scissors, paper}
 - Are all the same set

Properties of Sets

- Duplication is allowed
- {Sara, Sam, Frank, Julie, Sam, Frank}

Multi-sets

- $A \subseteq B$ means that all members of A are also members of B
- Read A is a subset (or equal to) of B
- Examples:
 - $\{1,2\} \subseteq \{1,2,7\}$
 - $\{a\} \subseteq \{\alpha \mid \alpha \text{ is a letter in the English alphabet}\}$
 - $\{H,I,T\} \subseteq \{H,I,T\}$
- For member elements, $H \in \{H,I,T\}$



Set Inclusion

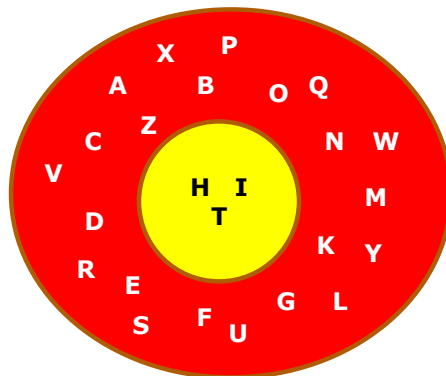
- A set is also a subset of itself
 $\{1,2,3\} \subseteq \{1,2,3\}$
- The empty set is also a subset of any set



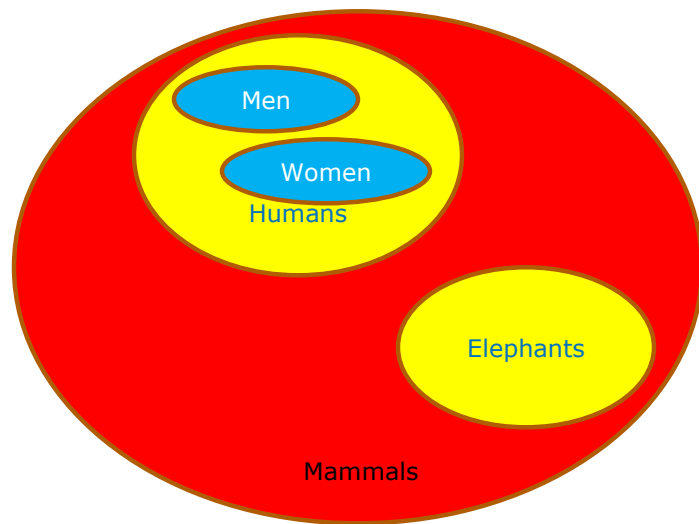
Set Inclusion: Subsets

- We write $x \in S$ to say that x is an element of the set S
- Examples:
 - $2 \in \{1, 2, 3\}$
 - $\$ \in \{\$, \text{¢}, \text{£}, \text{¥}, \text{€}, \text{€}\}$
 - $a \in \{a\}$
- This is **not** the same as “set inclusion!”

Set Membership



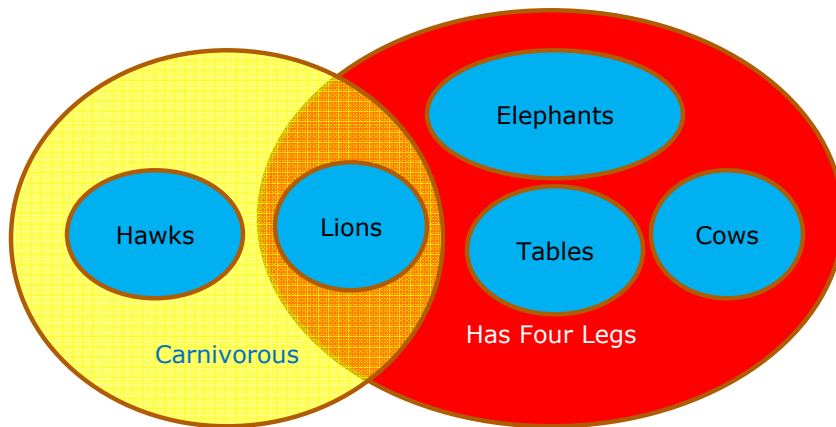
Set Inclusion – Venn Diagram



Set Inclusion – Venn Diagram

- Set Intersection
- $A \cap B$ = set of all elements that are in A **and** in B
- Example:
 - $A = \{1, 6, 8\}$, $B = \{1, 3, 5, 7\}$, $C = \{3, 5, 7\}$
 - $A \cap B = ???$
 - $B \cap C = ???$
 - $A \cap C = ???$
- We say sets S and T are *disjoint* if $S \cap T = \{\}$.

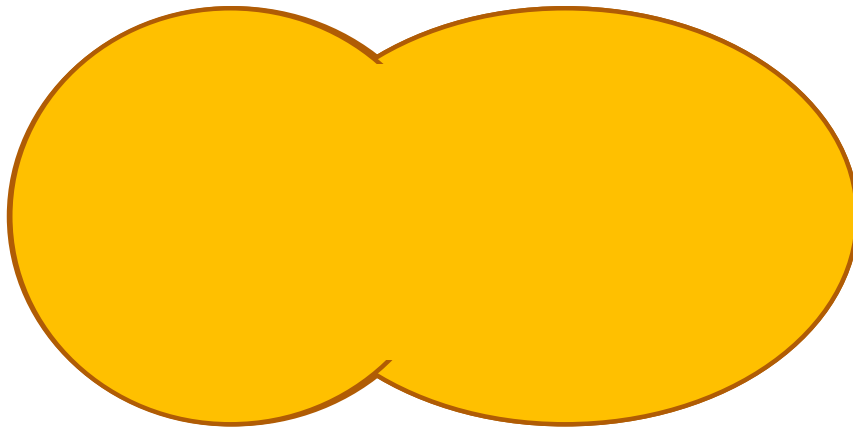
Operations on Sets



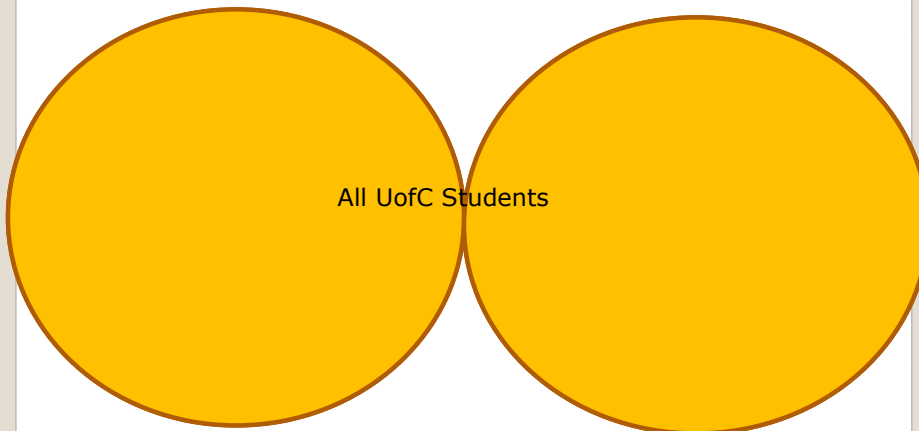
Intersection– Venn Diagram

- Set Union
- $A \cup B$ = set of all elements that are in A **or** in B
- Example:
 - $A = \{1, 6, 8\}$, $B = \{1, 3, 5, 7\}$, $C = \{3, 5, 7\}$
 - $A \cup B = ???$
 - $B \cup C = ???$
 - $A \cup C = ???$

Operations on Sets



Union- Venn Diagram



Union- Venn Diagram



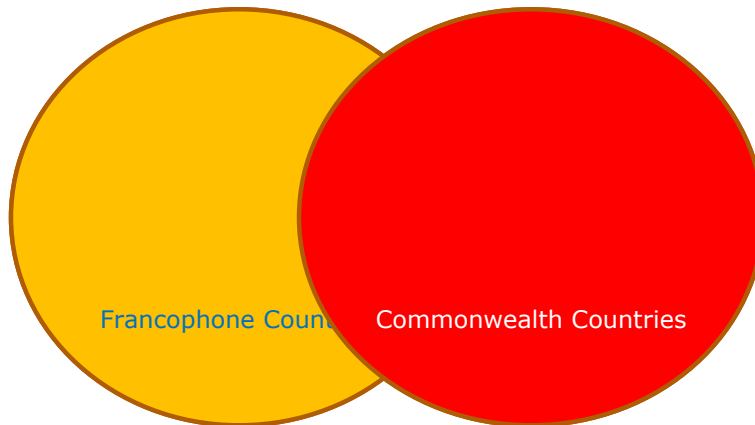
Francophone and Commonwealth Countries

Union– Venn Diagram

- Set Subtraction
- $A - B$ = set of all elements that are in A **but not** in B
- Example:
 - $A = \{1, 6, 8\}$, $B = \{1, 3, 5, 7\}$, $C = \{3, 5, 7\}$
 - $A - B = ???$
 - $B - C = ???$
 - $C - B = ???$
 - $C - A = ???$

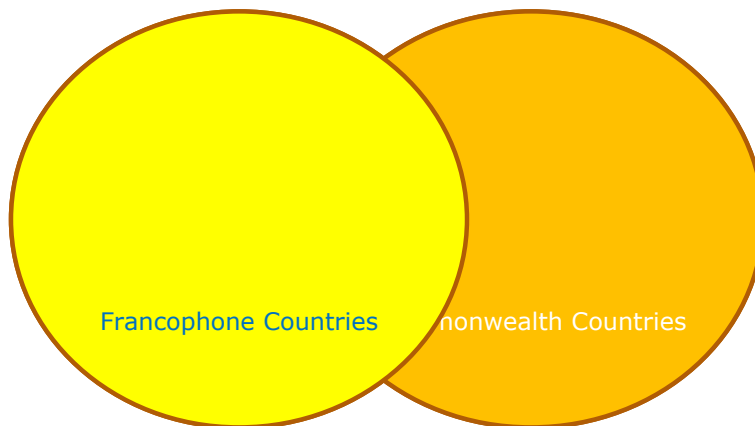
Operations on Sets

Francophone countries that are not members of the Commonwealth

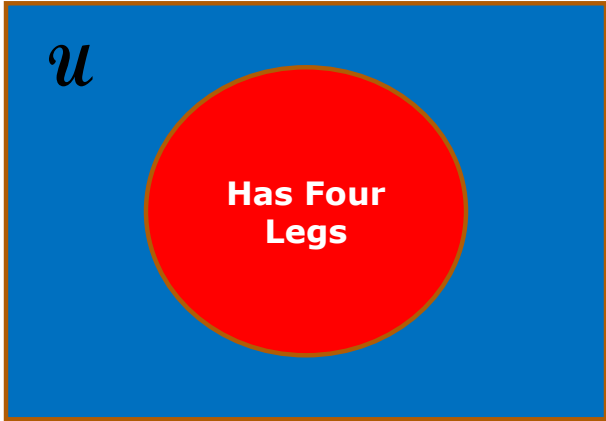


Subtraction- Venn Diagram

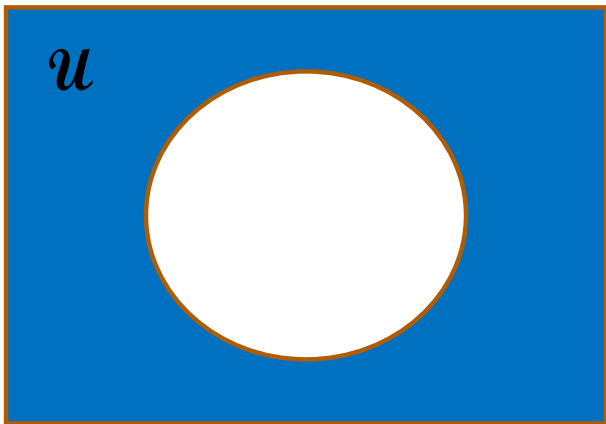
Commonwealth countries that are not Francophone



Subtraction- Venn Diagram



Complement



Complement

- Let A be a set
- $\overline{A} = \{x \mid x \text{ not } \in A\}$
- $A = \{x \mid x \text{ is male}\}$
- $\overline{A} = \{x \mid x \text{ is not male}\}$

Complement

- We use curly brackets (braces) to represent sets
 - $\{\text{rock, paper}\} = \{\text{paper, rock}\}$
- If order is important we use ordered tuples
 - $(\text{rock, paper}) \neq (\text{paper, rock})$
 - Maybe meaning (*choice of player 1, choice of player 2*)
- General form of a tuple: $(v_1, v_2, v_3, \dots, v_n)$
- In tuples, repetition of elements is allowed: $(1, 2, 1, 3)$

Ordered Tuples

- $A \times B = \{(a,b) \mid a \text{ is in } A \text{ and } b \text{ is in } B\}$
- Example: $A = \{a\}, B = \{1, 2\}$
- $A \times B = \{(a,1), (a,2)\}$

- In general $A_1 \times A_2 \times \dots \times A_n = \{(a_1, a_2, \dots, a_n) \mid a_1 \text{ is in } A_1 \text{ and } a_2 \text{ is in } A_2 \dots a_n \text{ is in } A_n\}$



Set Multiplication