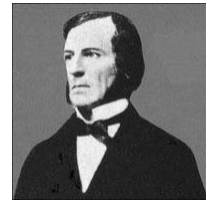


- 1815-1864
- English Mathematician
- His *The Mathematical Analysis of Logic*, 1848 is the first contribution to symbolic logic
- In this book he introduced what is today called Boolean Logic (or Algebra)
 - JT: Boolean (True, False outcome)



George Boole

Peeking into Computer Science

© Jalal Kawash 2010

1

Logic

Peeking into Computer Science



© Jalal Kawash 2010

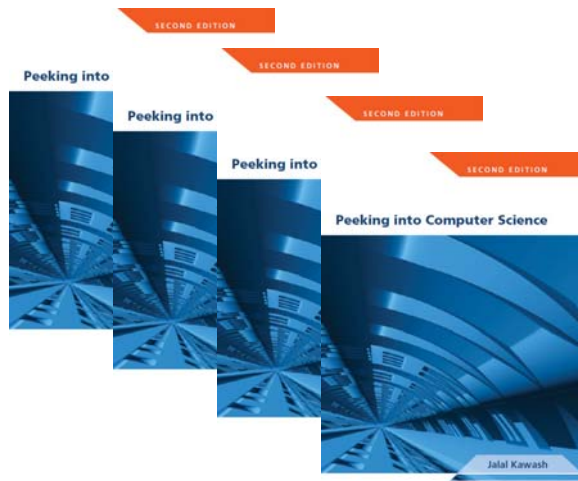
- Mandatory: Chapter 2 – Section 2.2

Reading Assignment

Peeking into Computer Science

© Jalal Kawash 2010

3



Predicate Logic

4

By the end of this section, you will be able to:

1. Define a predicate
2. Understand universal and existential quantifiers
3. Use quantification to convert a predicate to a proposition
4. Work with quantifier equivalence rules

Objectives

- Proposition: a declarative sentence that is either true or false, but not both.¹

- Example propositions

- $3 > 4$
 - $4 > 3$
- } JT: True/False clear cut

1) "Peeking into Computer Science" (2nd Ed) Kawash J.

JT's Extra: Review

- Predicate: a proposition where the value of a variable is unknown.
 - Example predicate
 - $P(X): X > 0$ } JT: True/False "it depends"

JT's Extra: New Material

- $X > 3$
 - Is not a proposition
- X is taller than Y
 - Is not a proposition
- These are predicates
- $P(X): X > 3$
- $Q(X,Y): X$ is taller than Y

Predicate Logic

- Predicates can be made propositions by

1. Substituting values for the variables

- $P(X): X > 3$, $P(4)$ is true, $P(-1)$ is false
- $Q(X,Y): X$ is taller than Y , $Q(\text{Debra}, \text{Doug})$

OR

2. Binding the variable with a quantifier

- Universal Quantifier $\forall x P(x)$
 - $P(x)$ is true for all x in the universe of discourse
- Existential Quantifier $\exists x P(x)$
 - $P(x)$ is true for at least one x in the universe of discourse

Quantification

- Universe of discourse: this 203 class
- $P(x): x$ is female

- $\forall x P(x) :$

- All students in this class are female

- $\exists x P(x)$

- There is at least one student in this class who is female

Quantification Examples

- Universe of discourse: all earth creatures
- $M(x)$: x is a monkey
- $F(x)$: x lives in a forest
- Express: some monkeys live in forests
- $\exists x (M(x) \wedge F(x))$:
 - At least some monkey lives in a forest



Quantification Examples

- Universe of discourse: all earth creatures
- $M(x)$: x is a monkey
- $F(x)$: x lives in a forest
- Express: all monkeys live in forests
- $\forall x (M(x) \wedge F(x))$: **X**
 - All earth creatures are monkeys and live in forests
- $\forall x (M(x) \rightarrow F(x))$:
 - From all creatures if x is a monkey, then x lives in a forest



Quantification Examples

- $\forall x P(x)$ is equivalent to $\neg [\exists x \neg P(x)]$
 - All monkeys are black
 - There is no one monkey which is not black
- $\exists x P(x)$ is equivalent to $\neg [\forall x \neg P(x)]$
 - There is at least one student who likes the course
 - It is not the case that all students do not like the course



Quantifier Equivalence