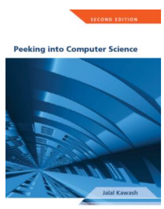
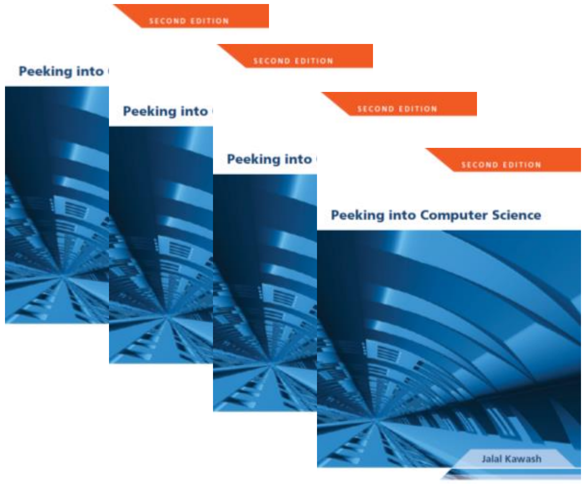
Elementary Set Theory

Peeking into Computer Science



Reading Assignment

- Mandatory: Chapter 2 – Sections 2.3 and 2.4



Sets and Set Operations

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At the end of this section, the you will be able to:

1. Understand the two basic properties of sets
2. Understand the (one) difference between sets and multisets
3. Define a set and use two types of set representations
4. Perform all 4 operations on sets
5. Use Venn diagrams to depict sets and operations on them
6. Define tuples and differentiate them from sets
7. Perform multiplication on sets (Cartesian products)

Objectives

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- A **set** is a collection of objects;
  - we call these objects the *elements* of the set.
- Examples:
  - Students in this class form (ie, are elements of) a set
  - The set of all positive integers
  - The set of all of your shirts
- Sometimes we call an element of a set a *member* of that set, instead.

Sets

- Small sets can be represented by listing its members
- A = {paper, scissors, rock}
  - All possible choices in the game
- B = {pawn, rook, knight, bishop, queen, king}
  - All chess pieces
- C = {1, 2, 3, 4, 5}
  - All positive integers less than or equal to 5

Representing Small Sets

Representing Big Sets

- May not be possible to list all members
- Some sets are infinite

- A = {x | x is a current student at UofC}
  - All current students at UofC

- B = {x | x is an even number}
  - Infinite set

The Empty Set

- An empty set contains no elements.
- Notation:

  A = { }

  A = ϕ

Properties of Sets

- Duplicates are not allowed (or do not count)
  - {paper, scissors, rock} is the same as {paper, scissors, rock, rock}
  - We should not repeat elements
- Order of members is irrelevant
  - {paper, scissors, rock}
  - {scissors, rock, paper}
  - {rock, scissors, paper}
  - Are all the same set

Multi-sets

- Duplication is allowed
- {Sara, Sam, Frank, Julie, Sam, Frank}

- $A \subseteq B$ means that all members of A are also members of B
- Read A is a subset (or equal to) of B
- Examples:

- $\{1,2\} \subseteq \{1,2,7\}$
- $\{a\} \subseteq \{\alpha \mid \alpha$ is a letter in the English alphabet$\}$
- $\{H,I,T\} \subseteq \{H,I,T\}$

- For member elements, $H \in \{H,I,T\}$

Set Inclusion (JT: Subset)

- A set is also a subset of itself

  $\{1,2,3\} \subseteq \{1,2,3\}$
- The empty set is also a subset of any set

Set Inclusion: Subsets

- We write x ∈ S to say that x is an element of the set S
- Examples:
  - 2 ∈{1, 2, 3}
  - $ ∈{$, ¢, £, ¥, €, ₠}
  - a ∈ {a}
- This is **not** the same as “set inclusion!”

Set Membership

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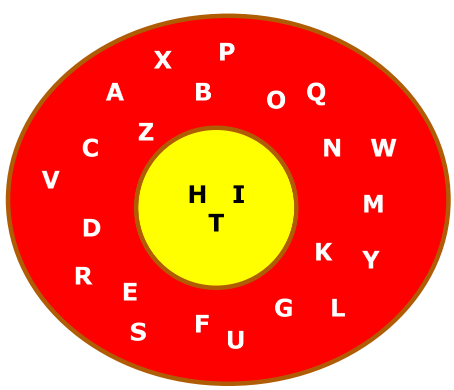

Set Inclusion – Venn Diagram

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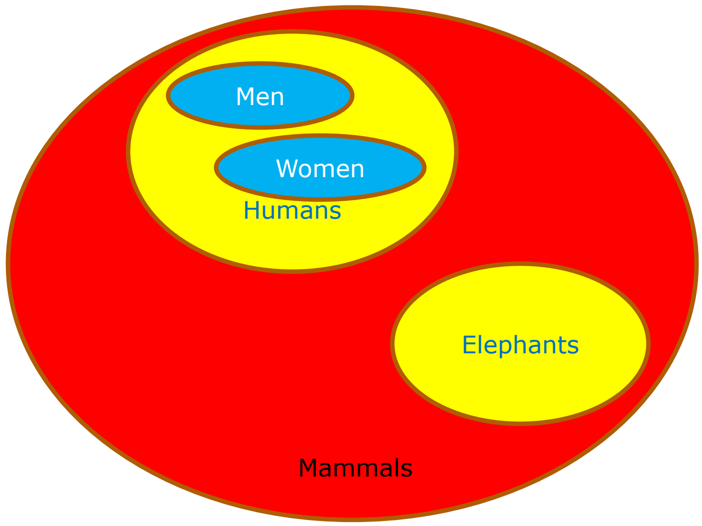
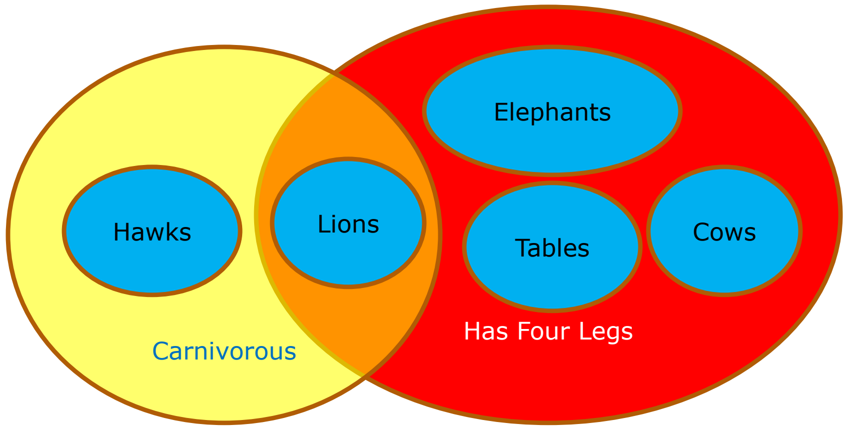


Set Inclusion – Venn Diagram

Operations on Sets

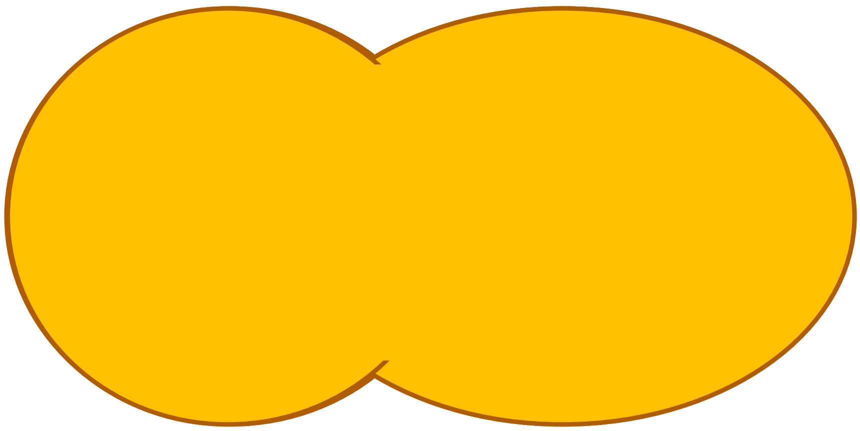
- Set Intersection
- A ∩ B = set of all elements that are in A **and** in B
- Example:
  - A = {1, 6, 8}, B = {1, 3, 5, 7}, C = {3, 5, 7}
  - A ∩ B = ???
  - B ∩ C = ???
  - A ∩ C = ???
- We say sets S and T are *disjoint* if S∩T={}.



Intersection– Venn Diagram

Operations on Sets

- Set Union
- A ∪ B = set of all elements that are in A **or** in B
- Example:
  - A = {1, 6, 8}, B = {1, 3, 5, 7}, C = {3, 5, 7}
  - A ∪ B =???
  - B ∪ C =???
  - A ∪ C =???



Union– Venn Diagram

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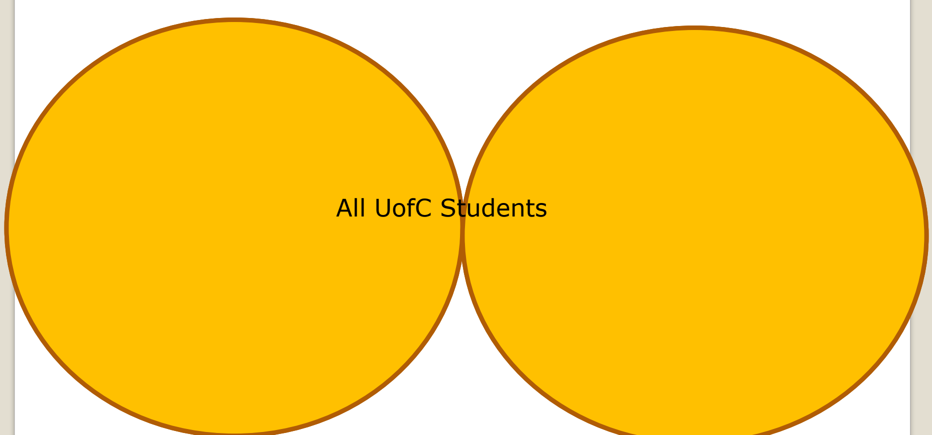

Union– Venn Diagram

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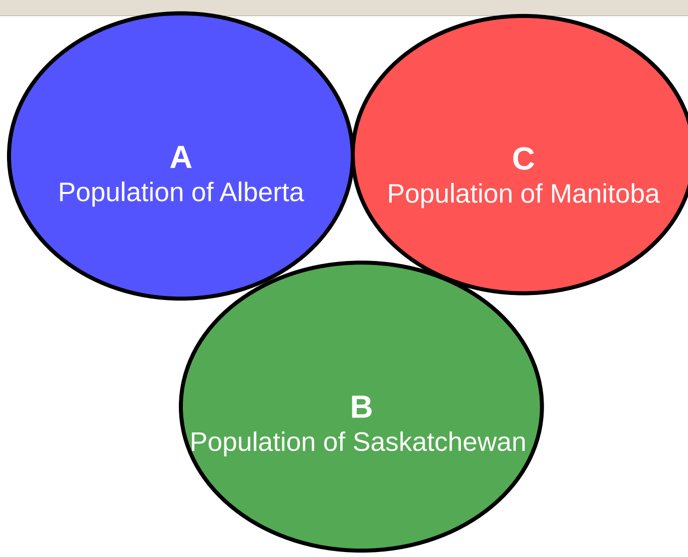
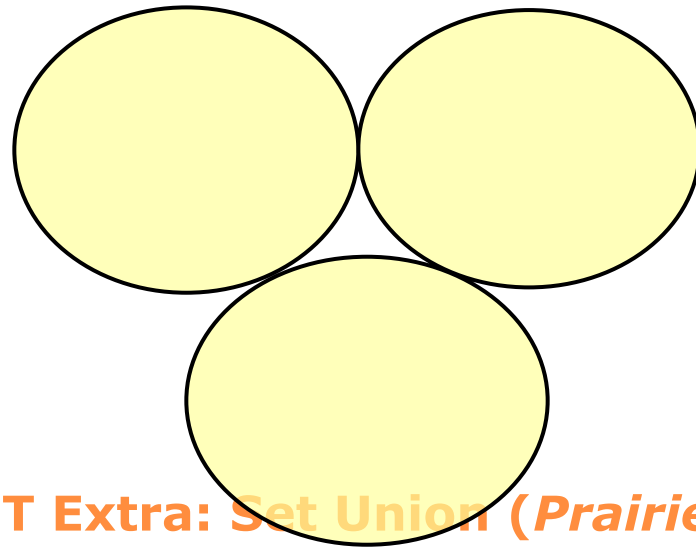
Union– Venn Diagram



JT Extra: Prairie Provinces

$A \cup B \cup C = D$ (Population of the Prairie provinces)



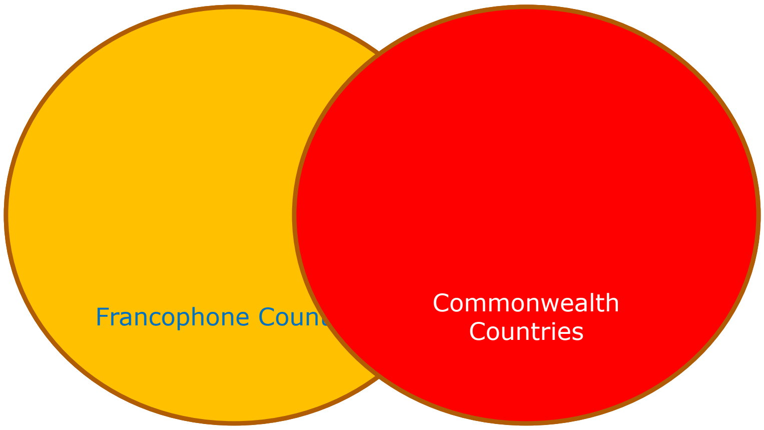
JT Extra: Set Union (*Prairies*)

Operations on Sets

- Set Subtraction
- A – B = set of all elements that are in A **but not** in B

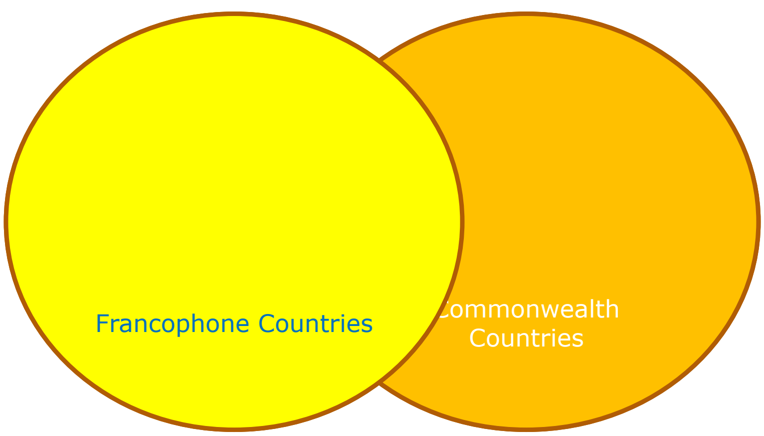
- Example:
  - A = {1, 6, 8}, B = {1, 3, 5, 7}, C = {3, 5, 7}
  - A – B = ???
  - B – C = ???
  - C – B = ???
  - C – A = ???

Francophone countries that are not members of the Commonwealth

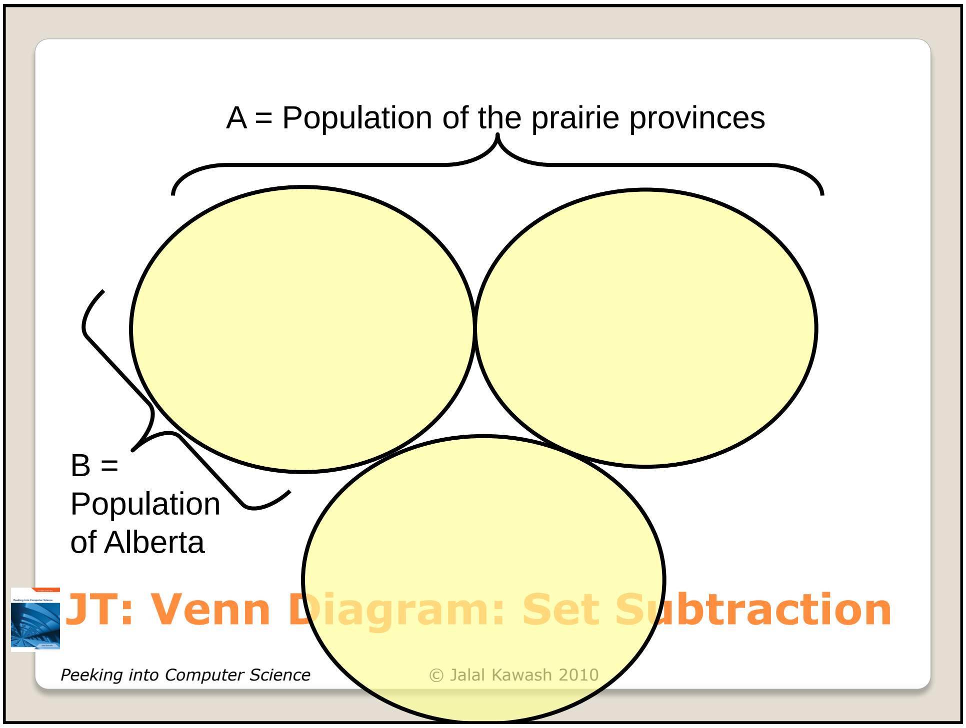


Subtraction– Venn Diagram

Commonwealth countries that are not Francophone



Subtraction– Venn Diagram


A = Population of the prairie provinces
B = Population of Alberta
JT: Venn Diagram: Set Subtraction
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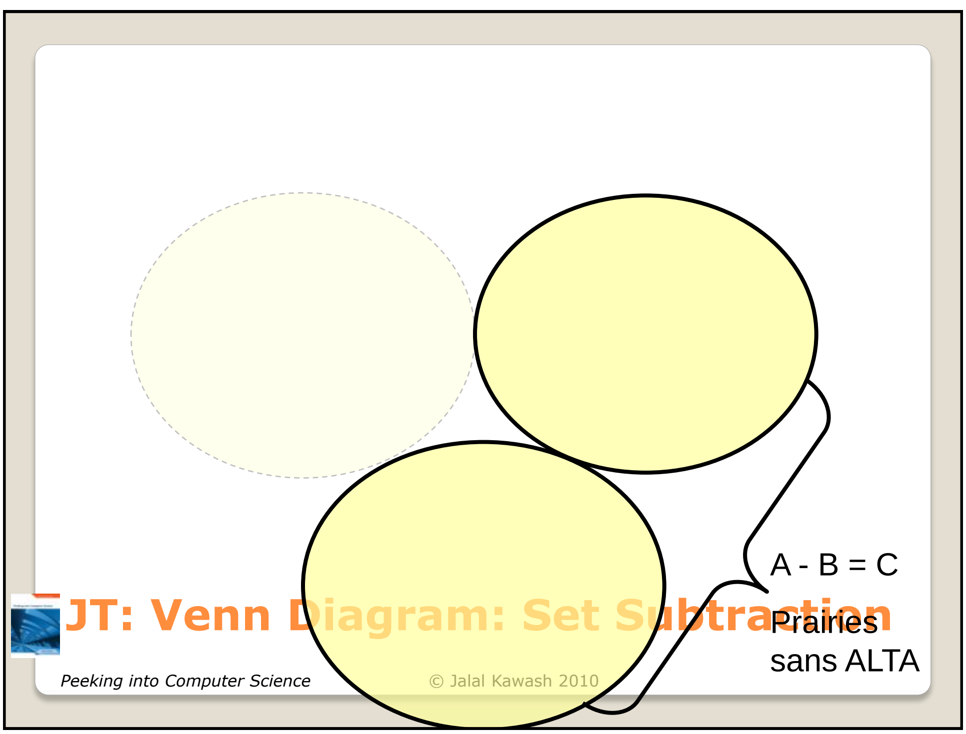
A - B = C
Prairies sans ALTA
JT: Venn Diagram: Set Subtraction
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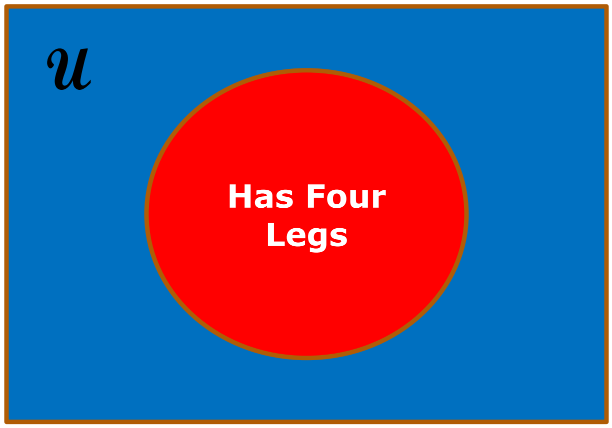
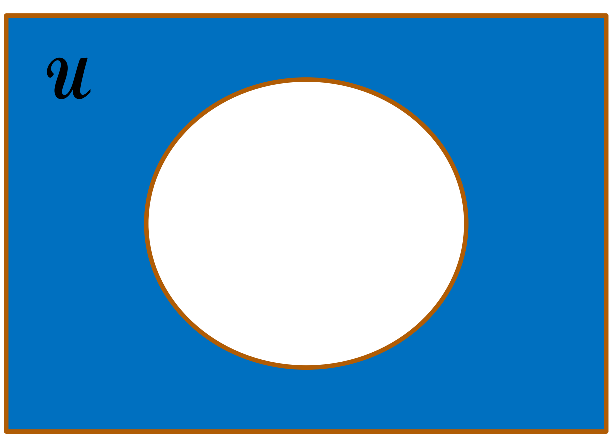


Complement



Complement

Complement

- Let A be a set
- $\overline{A}$ = {x | x not ∈ A}

- A = { x | x is male}
- $\overline{A}$ = {x | x is not male}

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Ordered Tuples

- We use curly brackets (braces) to represent sets
  - {rock, paper} = {paper, rock}
- If order is important we use ordered tuples
  - (rock, paper) ≠ (paper, rock)
  - Maybe meaning (*choice of player 1*, *choice of player 2*)
- General form of a tuple: $(v_1, v_2, v_3, \ldots v_n)$
- In tuples, repetition of elements is allowed: (1, 2, 1, 3)

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- A x B = {(a,b) | a is in A and b is in B}
- Example: A = {a}, B = {1, 2}
- A x B = {(a,1), (a,2)}
- JT's extra:
  - "Takes all combinations from the sets"
  - The operation may be used in decision making to ensure that all combinations have been covered.
- In general A_1 x A_2 x … x A_n = {(a_1,a_2,…, a_n) | a_1 is in A_1 and a_2 is in A_2 … a_n is in A_n}

Set Multiplication

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- Developing a game where all combinations must be considered in order to determine the outcome.
- Each combination is a tuple (not a set).

  A = {player one, player two}
  B = {rock, paper, scissors}
  A x B = {(player one, rock), (player one, paper), (player one, scissors),
  (player two, rock), (player two, paper), (player two, scissors)}
- (Examples from actual software will be much more complex and taking a systematic approach helps ensure that nothing is missed).

  A = {player one, player two, player three...}
  B = {completed quest one, completed quest two...}
  C = {healthy, injured, poisoned, diseased, dead, gone forever}

JT's Extra: How Set Multiplication May Be Applied

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