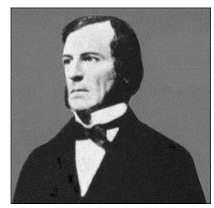
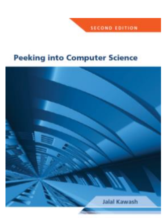
- 1815-1864
- English Mathematician
- His *The Mathematical Analysis of Logic*, 1848 is the first contribution to symbolic logic
- In this book he introduced what is today called Boolean Logic (or Algebra)
  - JT: Boolean (True, False outcome)



George Boole

Logic

Peeking into Computer Science



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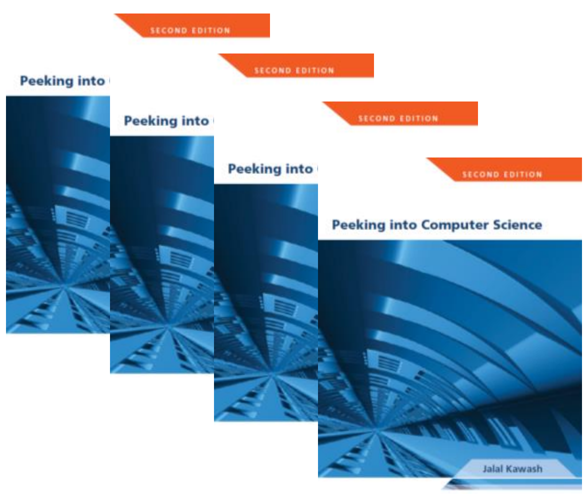
Reading Assignment

- Mandatory: Chapter 2 – Section 2.2



Predicate Logic

Objectives

By the end of this section, you will be able to:

1. Define a predicate
2. Understand universal and existential quantifiers
3. Use quantification to convert a predicate to a proposition
4. Work with quantifier equivalence rules

JT's Extra: Review

- Proposition: a declarative sentence that is either true or false, but not both.[1]
  - Example propositions
    - 3 > 4
    - 4 > 3

JT: True/False clear cut

1) "Peeking into Computer Science" (2nd Ed) Kawash J.

JT's Extra: New Material

- Predicate: a proposition where the value of a variable is unknown.
  - Example predicate
    - P(X): X > 0 } JT: True/False "it depends"

Predicate Logic

- X > 3
  - Is not a proposition
- X is taller than Y
  - Is not a proposition
- These are predicates
- P(X): X > 3
- Q(X,Y): X is taller than Y

Quantification

- Predicates can be made propositions by
  1. Substituting values for the variables
     - P(X): X > 3, P(4) is true, P(-1) is false
     - Q(X,Y): X is taller than Y, Q(Debra, Doug)

  OR

  2. Binding the variable with a quantifier in the universe of discourse[1]
     - Universal Quantifier $\forall x\, P(x)$
       - P(x) is true for all x in the universe of discourse
     - Existential Quantifier $\exists x\, P(x)$
       - P(x) is true for at least one x in the universe of discourse

1 JT: Universe of discourse = "…the entities referred to in a discourse or an argument." – www.thefreedictionary.com

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JT's Extra: Additional Example

- Predicate:
  - P(x,y): x works as a y.
- Proposition:
  - P(Jet Li, actor): Jet Li works as an actor.

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JT's Extra: Additional Example

- Predicate:
  - P(x): x is odd.
- Proposition:
  - P(3): 3 is odd

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Quantification Examples

- Universe of discourse: this 203 class
- P(x): x is female
- $\forall x\ P(x)$
  - All students in this class are female
- $\exists x\ P(x)$
  - There is at least one student in this class who is female

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- **Existential Quantifier** and **Connective OR**
  - If all the elements in the universe of discourse can be listed, then the existential quantification *xP(x)* is equivalent to the disjunction: ***P(x1) P(x2) P(x3) … P(xn)***

JT's Extra: "There Exists" (at least one)

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- **Universal Quantifier** and **Connective AND**
  - If all the elements in the universe of discourse can be listed then the universal quantification *x P(x)* is equivalent to the conjunction: ***P(x1)*) *P(x2) P(x3) … P(xn)*** .

JT's Extra: "For All" (Applies in every case)

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JT's Extra

- Universe of discourse: cars
- W(x): has wheels
  - All cars have wheels
  - Some cars have wheels

Quantification Examples

- Universe of discourse: all earth creatures
- M(x): x is a monkey
- F(x): x lives in a forest
- Express: some monkeys live in forests
- $\exists x\ (M(x) \wedge F(x))$:
  - At least some monkey lives in a forest

Quantification Examples

- Universe of discourse: all earth creatures
- $M(x)$: x is a monkey
- $F(x)$: x lives in a forest

- Express: all monkeys live in forests
- $\forall x\ (M(x) \wedge F(x))$: X
  - All earth creatures are monkeys and live in forests
- $\forall x\ (M(x) \rightarrow F(x))$:
  - From all creatures if x is a monkey, then x lives in a forest

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JT's Extra: Additional Examples

- Universe of discourse: people
- $T(X)$: x is tall
- Contrast:
  - $\forall x\ (T(x) \wedge \neg T(x))$
  - $\exists x\ (T(x) \wedge \neg T(x))$
  - $\forall x\ (T(x) \vee \neg T(x))$
  - $\exists x\ (T(x) \vee \neg T(x))$

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- $\forall x\ P(x)$ is equivalent to $\neg\ [\exists x\ \ \neg\ P(x)]$
  - All monkeys are black
  - There is no one monkey which is not black
- $\exists x\ P(x)$ is equivalent to $\neg\ [\forall x\ \ \neg\ P(x)]$
  - There is at least one student who likes the course
  - It is not the case that all students do not like the course

Quantifier Equivalence

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