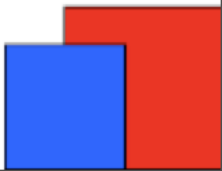




Translating English to FOL

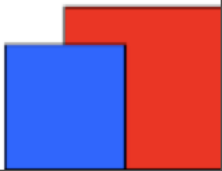
- Deb is not tall.





Translating English to FOL

- Every gardener likes the sun.





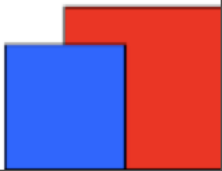
Translating English to FOL

- You can fool some of the people all of the time.
- 



Translating English to FOL

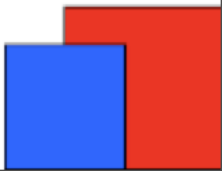
- You can fool all of the people some of the time.





Translating English to FOL

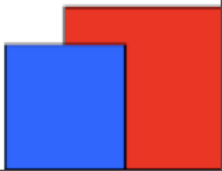
- All purple mushrooms are poisonous.





Translating English to FOL

- No purple mushroom is poisonous.





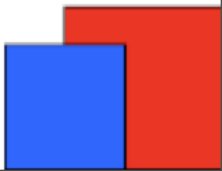
Translating English to FOL

- There are exactly two purple mushrooms.
- 



Translating English to FOL

- X is above Y, if X is directly on top of Y, or else there is a pile of one or more other objects directly on top of one another starting with X and ending with Y.





Does Ziggy eat fish?





Generalized Modus Ponens with Horn Clauses

Forward Chaining:

1. $(\forall x) \text{ cat}(x) \Rightarrow \text{likes}(x, \text{Fish})$
 2. $(\forall x) (\forall y) (\text{cat}(x) \wedge \text{likes}(x,y) \Rightarrow \text{eats}(x,y)$
 3. $\text{cat}(\text{Ziggy})$
 4. (1), (3) $\rightarrow \text{likes}(\text{Ziggy}, \text{Fish})$
 5. (3), (4), (2) $\rightarrow \text{eats}(\text{Ziggy}, \text{Fish})$
- 



Generalized Modus Ponens with Horn Clauses

Backward Chaining:

1. $(\forall x) \text{ cat}(x) \Rightarrow \text{likes}(x, \text{Fish})$
 2. $(\forall x) (\forall y) (\text{cat}(x) \wedge \text{likes}(x,y) \Rightarrow \text{eats}(x,y)$
 3. $\text{cat}(\text{Ziggy})$
- Goal: $\text{eats}(\text{Ziggy}, \text{Fish})$ – (2) has $\text{eats}(x,y)$ so show:
 - $\text{cat}(\text{Ziggy})$ and $\text{likes}(\text{Ziggy}, \text{Fish})$
- 



Generalized Modus Ponens with Horn Clauses

- `cat(Ziggy)` – axiom (3) – ‘solved’
 - `likes(Ziggy, Fish)` – (1) has `likes(x, Fish)` so show
 - `cat(Ziggy)`
 - `cat(Ziggy)` – axiom (3) again – ‘solved’
- 



Rules for Converting FOL wffs to clauses

1. Eliminate $\Leftrightarrow$; replace
 $P \Leftrightarrow Q$ with $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$
 2. Eliminate $\Rightarrow$; replace
 $P \Rightarrow Q$ with $\neg P \vee Q$
 3. Reduce the scope of $\neg$; replace
 - $\neg \neg P$ with P
 - $\neg(P \vee Q)$ with $\neg P \wedge \neg Q$
 - $\neg(P \wedge Q)$ with $\neg P \vee \neg Q$
 - $\neg \forall x P$ with $\exists x \neg P$
 - $\neg \exists x P$ with $\forall x \neg P$
- 



Rules for Converting FOL wffs to clauses

4. Standardize Variables; give each quantified variable its own unique name
eg. $\forall x(P(x) \vee (\exists Q(x)))$ with $\forall xP(x) \vee (\exists yQ(y))$
 5. Eliminate Existential Quantifiers
 6. Eliminate Universal Quantifiers
 7. Distribute $\wedge$ over $\vee$; replace
 $(P \wedge Q) \vee R$ with $(P \vee R) \wedge (Q \vee R)$
 $(P \vee Q) \vee R$ with $(P \vee Q \vee R)$
- 



Rules for Converting FOL wffs to clauses

8. Create separate clauses; replace $(P(x) \wedge Q(x))$ with $\{P(x), Q(x)\}$
 9. Standardize variables apart again so that each clause contains variables names that do not occur in any other clause;
- 



Converting to CNF

- $(\forall x) (P(x) \Rightarrow ((\forall y) (P(y) \Rightarrow P(f(x,y))) \wedge \neg(\forall y) (Q(x,y) \Rightarrow P(y))))$

Converting to CNF

1) Eliminate $\Leftrightarrow$; replace
 $P \Leftrightarrow Q$ with $(P \Rightarrow Q) \wedge$
 $(Q \Rightarrow P)$

- $(\forall x) (P(x) \Rightarrow ((\forall y) (P(y) \Rightarrow P(f(x,y))) \wedge \neg(\forall y) (Q(x,y) \Rightarrow P(y))))$

Converting to CNF

2) Eliminate $\Rightarrow$; replace
 $P \Rightarrow Q$ with $\neg P \vee Q$

- $(\forall x) (P(x) \Rightarrow ((\forall y) (P(y) \Rightarrow P(f(x,y))) \wedge \neg(\forall y) (Q(x,y) \Rightarrow P(y))))$



- $(\forall x) (\neg P(x) \vee ((\forall y) (\neg P(y) \vee P(f(x,y))) \wedge \neg(\forall y) (\neg Q(x,y) \vee P(y))))$

Converting to CNF

3) Reduce the scope of $\neg$;

- $(\forall x) (\neg P(x) \vee ((\forall y) (\neg P(y) \vee P(f(x,y))) \wedge \neg(\forall y) (\neg Q(x,y) \vee P(y))))$



- $(\forall x) (\neg P(x) \vee ((\forall y) (\neg P(y) \vee P(f(x,y))) \wedge (\exists y) (Q(x,y) \vee \neg P(y))))$

Converting to CNF

4) Standardize Variables

- $(\forall x) (\neg P(x) \vee ((\forall y) (\neg P(y) \vee P(f(x,y))) \wedge (\exists y) (Q(x,y) \vee \neg P(y))))$



- $(\forall x) (\neg P(x) \vee ((\forall y) (\neg P(y) \vee P(f(x,y))) \wedge (\exists z) (Q(x,z) \vee \neg P(z))))$

Converting to CNF

5) Eliminate Existential Quantifiers

- $(\forall x) (\neg P(x) \vee ((\forall y) (\neg P(y) \vee P(f(x,y)))) \wedge (\exists z) (Q(x,z) \vee \neg P(z)))$



- $(\forall x) (\neg P(x) \vee ((\forall y) (\neg P(y) \vee P(f(x,y)))) \wedge (Q(x,g(x)) \vee \neg P(g(x))))$

Converting to CNF

6) Eliminate Universal Quantifiers

- $(\forall x) (\neg P(x) \vee ((\forall y) (\neg P(y) \vee P(f(x,y))) \wedge (Q(x,g(x)) \vee \neg P(g(x)))))$



- $(\neg P(x) \vee ((\neg P(y) \vee P(f(x,y))) \wedge (Q(x,g(x)) \vee \neg P(g(x)))))$

Converting to CNF

7) Distribute $\wedge$ over $\vee$

- $(\neg P(x) \vee ((\neg P(y) \vee P(f(x,y))) \wedge (Q(x,g(x)) \vee \neg P(g(x))))$



- $(\neg P(x) \vee \neg P(y) \vee P(f(x,y))) \wedge (\neg P(x) \vee Q(x,g(x))) \wedge (\neg P(x) \vee \neg P(g(x)))$

Converting to CNF

8) Create separate clauses

- $(\neg P(x) \vee \neg P(y) \vee P(f(x,y))) \wedge (\neg P(x) \vee Q(x,g(x))) \wedge (\neg P(x) \vee \neg P(g(x)))$



- $\neg P(x) \vee \neg P(y) \vee P(f(x,y))$
- $\neg P(x) \vee Q(x,g(x))$
- $\neg P(x) \vee \neg P(g(x))$

Converting to CNF

9) Standardize variables

- $\neg P(x) \vee \neg P(y) \vee P(f(x,y))$
- $\neg P(x) \vee Q(x,g(x))$
- $\neg P(x) \vee \neg P(g(x))$



- $\neg P(x) \vee \neg P(y) \vee P(f(x,y))$
- $\neg P(z) \vee Q(x,g(z))$
- $\neg P(w) \vee \neg P(g(w))$



Mountain People!

- Tom, Bob and Nancy are all members of the Alpine Club of Canada. Every member of the Alpine Club is either a skier or a climber or both. No climber likes rain and all skiers like snow. Nancy dislikes whatever Tom likes and likes whatever Tom dislikes. Tom likes rain and snow.
 - Is there a member of the AAC who is a climber but not a skier.
- 



Mountain People - Predicates

- $\text{Skier}(x)$ – x is a skier, the domain of x is ACC members
 - $\text{Climber}(x)$ – x is a climber, the domain of x is ACC members
 - $\text{Likes}(x,y)$ – x likes y , the domain of x is AAC members and the domain of y is $\{\text{Rain, Snow}\}$
- 

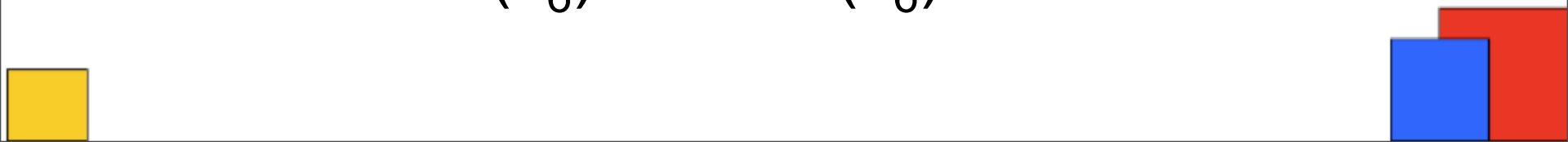


Mountain People - WFFs

1. $\forall x \text{ Skier}(x) \vee \text{Climber}(x)$
 2. $\neg \exists x \text{ Climber}(x) \wedge \text{Likes}(x, \text{Rain})$
 3. $\forall x \text{ Skier}(x) \Rightarrow \text{Likes}(x, \text{Snow})$
 4. $\forall y \text{ Likes}(\text{Nancy}, y) \Leftrightarrow \neg \text{Likes}(\text{Tom}, y)$
 5. $\text{Likes}(\text{Tom}, \text{Rain}) \wedge \text{Likes}(\text{Tom}, \text{Snow})$
 6. $\exists x \text{ Climber}(x) \wedge \neg \text{Skier}(x)$ // This is what we want to know.
- 



Mountain People - Clauses

1. $\text{Skier}(x_1) \vee \text{Climber}(x_1)$
 2. $\neg \text{Climber}(x_2) \vee \neg \text{Likes}(x_2, \text{Rain})$
 3. $\neg \text{Skier}(x_3) \vee \text{Likes}(x_3, \text{Snow})$
 4. $\neg \text{Likes}(\text{Tom}, x_4) \vee \neg \text{Likes}(\text{Nancy}, x_4)$
 5. $\text{Likes}(\text{Tom}, x_5) \vee \text{Likes}(\text{Nancy}, x_5)$
 6. $\text{Likes}(\text{Tom}, \text{Rain})$
 7. $\text{Likes}(\text{Tom}, \text{Snow})$
 8. $\neg \text{Climber}(x_6) \vee \text{Skier}(x_6)$
- 



Mountain People Resolution

- 1) $\text{Skier}(x_1) \vee \text{Climber}(x_1)$ and
8) $\neg \text{Climber}(x_6) \vee \text{Skier}(x_6)$ produces:
 - 9) $\text{Skier}(x_1) \ominus = \{x_6/x_1\}$
 - 9) $\text{Skier}(x_1)$ and
3) $\neg \text{Skier}(x_3) \vee \text{Likes}(x_3, \text{Snow})$ produces:
 - 10) $\text{Likes}(x_1, \text{Snow}) \ominus = \{x_3/x_1\}$
- 



Mountain People Resolution

- 10) Likes(x_1 , Snow) and
4) $\neg$ Likes(Tom, x_4) $\vee$ $\neg$ Likes(Nancy, x_4)
produces:
 - 11) $\neg$ Likes(Tom, Snow) $\Theta = \{x_4/\text{Snow}, x_1/\text{Nancy}\}$
- 11) $\neg$ Likes(Tom, Snow) and
7) Likes(Tom, Snow) produces
 - 12) $\square$

