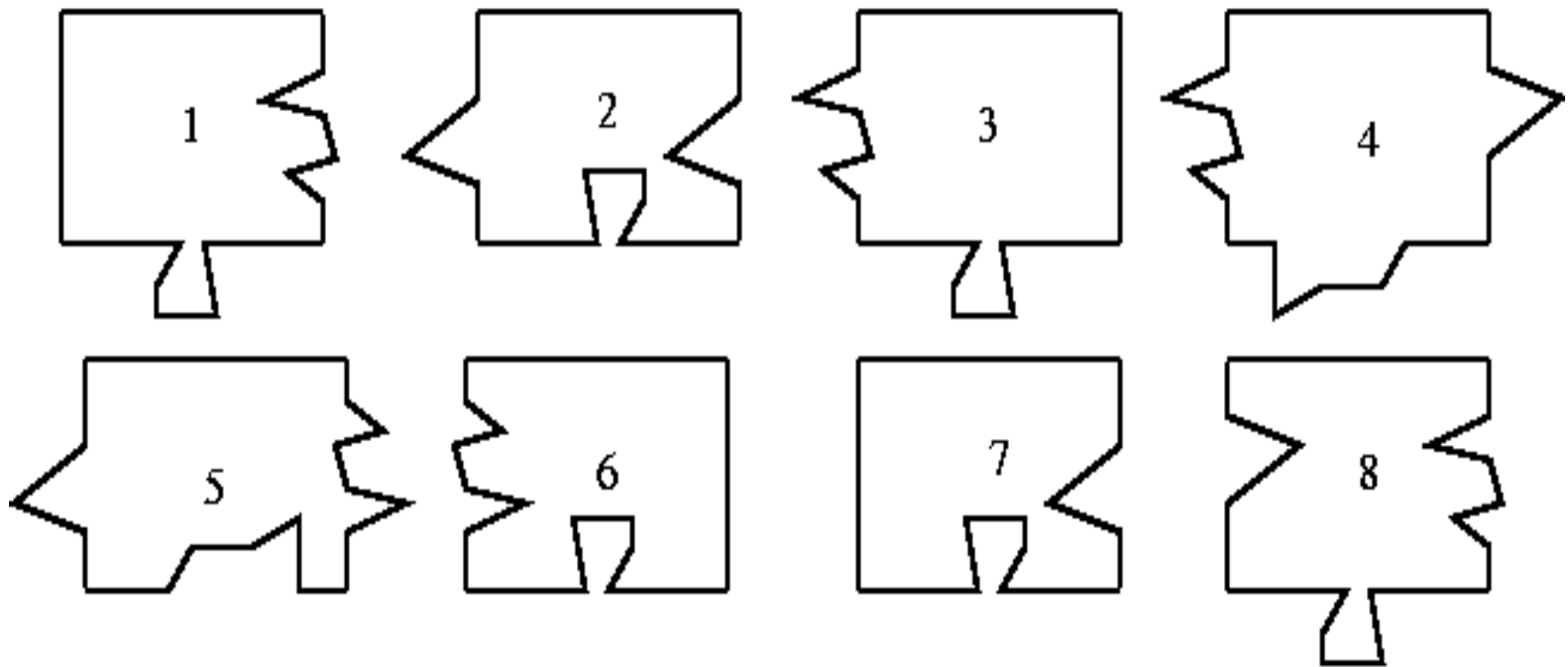


The Puzzle



Search Control 2

- $f_{\text{field},2}$ – choose a field on the matrix that is connected to the existing pieces and has the smallest number of options for pieces to place
- $f_{\text{piece},2}$ – choose a piece that ‘fits’ in the field chosen
- $f_{\text{remove},2}$ – choose a field that has a piece that can be removed without breaking up the puzzle

Search Control 2

$$S_0 = \left(\begin{array}{c|c|c|c} & \overset{A_0}{\downarrow} & & \overset{Rest_0}{\downarrow} \\ \hline (0,-) & (0,-) & (0,-) & (0,-) \\ \hline (0,-) & (0,-) & (0,-) & (0,-) \end{array} , \{1, 2, 3, 4, 5, 6, 7, 8\} \right)$$

$$f_{\text{field},2}(s_0) = (1, 1)$$

$$f_{\text{piece},2}((A_0, Rest_0), 1, 1) = (1, \uparrow), (3, \rightarrow), (6, \rightarrow), (7, \uparrow)$$

Search Control 2

$$S_1 = \left(\begin{array}{c|c|c|c} & \textcolor{violet}{A_1} \downarrow & & \textcolor{violet}{Rest_1} \downarrow \\ \hline (7, \uparrow) & (0, -) & (0, -) & (0, -) \\ \hline (0, -) & (0, -) & (0, -) & (0, -) \end{array} \right\{1, 2, 3, 4, 5, 6, 8\}$$

$f_{\text{field},2}(s_1) = (2, 1)$ ((2,1) has 1 fitting piece, (1,2) has 2 pieces (2 or 5))

$f_{\text{piece},2}((A_1, \text{Rest}_1), 2, 1) = (3, \downarrow)$

Search Control 2

$$S_2 = \left(\begin{array}{c|c|c|c} & \overset{A_2}{\downarrow} & & \overset{\text{Rest}_2}{\downarrow} \\ \hline (7, \uparrow) & (0, -) & (0, -) & (0, -) \\ \hline (3, \downarrow) & (0, -) & (0, -) & (0, -) \end{array} \right) \{1, 2, 4, 5, 6, 8\}$$

$$f_{\text{field},2}(s_2) = (1, 2)$$

$$f_{\text{piece},2}((A_2, \text{Rest}_2), 1, 2) = (5, \uparrow) \text{ or } (2, \uparrow)$$

Search Control 2

$$S_3 = \left(\begin{array}{|c|c|c|c|} \hline (7, \uparrow) & (5, \uparrow) & (0, -) & (0, -) \\ \hline (3, \downarrow) & (0, -) & (0, -) & (0, -) \\ \hline \end{array} \right) \{1, 2, 4, 6, 8\}$$

A_3 $Rest_3$
 $\downarrow$ $\downarrow$

$$f_{\text{field},2}(s_3) = (1, 3)$$

$f_{\text{piece},2}((A_3, Rest_3), 1, 3) = (6, \uparrow)$ – no other options!

Search Control 2

$$S_4 = \left(\begin{array}{c|c|c|c} & \textcolor{violet}{A}_4 & & \\ \hline (7, \uparrow) & (5, \uparrow) & (6, \uparrow) & (0, -) \\ \hline (3, \downarrow) & (0, -) & (0, -) & (0, -) \\ \hline \end{array} \right\{1, 2, 4, 8\} \textcolor{violet}{Rest}_4 \right)$$

$$f_{\text{field},2}(s_4) = (2, 3)$$

$$f_{\text{piece},2}((A_4, \text{Rest}_4), 2, 3) = (8, \downarrow) \text{ or } (1, \downarrow)$$

Search Control 2

$$S_5 = \left(\begin{array}{c|c|c|c} & \textcolor{violet}{A}_5 & & \textcolor{violet}{Rest}_5 \\ \hline (7, \uparrow) & (5, \uparrow) & (6, \uparrow) & (0, -) \\ \hline (3, \downarrow) & (0, -) & (1, \downarrow) & (0, -) \end{array} \right), \{2, 4, 8\}$$

$$f_{\text{remove},2}((A_5, \text{Rest}_5)) = (2, 3) \text{ or } (2, 1)$$

Any other combination results in islands

Search Control 2

$$S_6 = \left(\begin{array}{c|c|c|c} & \textcolor{violet}{A}_6 & & \textcolor{violet}{Rest}_6 \\ \hline (7, \uparrow) & (5, \uparrow) & (6, \uparrow) & (0, -) \\ \hline (3, \downarrow) & (0, -) & (0, -) & (0, -) \end{array} \right) \{1, 2, 4, 8\}$$

$$f_{\text{field},2}(s_6) = (2, 3)$$

$$f_{\text{piece},2}((A_6, \text{Rest}_6), 2, 3) = (8, \downarrow)$$

Search Control 2

$$S_7 = \left(\begin{array}{c|c|c|c} & \textcolor{violet}{A_7} \downarrow & & \textcolor{violet}{Rest_7} \downarrow \\ \hline (7, \uparrow) & (5, \uparrow) & (6, \uparrow) & (0, -) \\ \hline (3, \downarrow) & (0, -) & (8, \downarrow) & (0, -) \\ \hline \end{array} \right)_{\{1, 2, 4\}}$$

$$f_{\text{remove},2}((A_7, \text{Rest}_7)) = (2, 3) \text{ or } (2, 1)$$

Any other combination results in islands

Search Control 2

$$S_8 = \left(\begin{array}{c} \textcolor{violet}{A_8} \downarrow \\ \begin{array}{|c|c|c|c|} \hline (7,\uparrow) & (5,\uparrow) & (6,\uparrow) & (0,-) \\ \hline (3,\downarrow) & (0,-) & (0,-) & (0,-) \\ \hline \end{array} \textcolor{violet}{Rest_8} \downarrow \end{array} \right) \{1, 2, 4, 8\}$$

$$f_{\text{remove},2}((A_8, \text{Rest}_8)) = (1, 3) \text{ or } (2, 1)$$

Any other combination results in islands

Search Control 2

$$S_9 = \left(\begin{array}{c} \textcolor{violet}{A_9} \downarrow \\ \begin{array}{|c|c|c|c|} \hline (7,\uparrow) & (5,\uparrow) & (0,-) & (0,-) \\ \hline (3,\downarrow) & (0,-) & (0,-) & (0,-) \\ \hline \end{array} \textcolor{violet}{\text{Rest}_9} \downarrow \end{array} \right) \{1, 2, 4, 6, 8\}$$

$$f_{\text{remove},2}((A_9, \text{Rest}_9)) = (1, 2) \text{ or } (2, 1)$$

Any other combination results in islands

Search Control 2

$$S_{10} = \left(\begin{array}{c} \begin{array}{c} A_{10} \\ \downarrow \end{array} \begin{array}{|c|c|c|c|} \hline (7, \uparrow) & (0, -) & (0, -) & (0, -) \\ \hline (3, \downarrow) & (0, -) & (0, -) & (0, -) \\ \hline \end{array} \begin{array}{c} \text{Rest}_{10} \\ \downarrow \end{array} \{1, 2, 4, 5, 6, 8\} \end{array} \right)$$

$$f_{\text{field},2}(s_{10}) = (1, 2)$$

$f_{\text{piece},2}((A_{10}, \text{Rest}_{10}), 1, 2) = (2, \uparrow)$ – since $(5, \uparrow)$ has all ready been tried

Search Control 2

$$S_{11} = \left(\begin{array}{|c|c|c|c|} \hline (7, \uparrow) & (2, \uparrow) & (0, -) & (0, -) \\ \hline (3, \downarrow) & (0, -) & (0, -) & (0, -) \\ \hline \end{array} \right) \{1, 4, 5, 6, 8\}$$

A_{11} points to the top-left cell $(7, \uparrow)$.
 $Rest_{11}$ points to the top-right cell $(0, -)$.

$$f_{\text{field},2}(s_{11}) = (1, 3)$$

$f_{\text{piece},2}((A_{11}, Rest_{11}), 1, 2) = (5, \uparrow)$ – again this is the only option now

Search Control 2

$$S_{12} = \left(\begin{array}{c} \begin{array}{c} A_{12} \\ \downarrow \end{array} \begin{array}{|c|c|c|c|} \hline (7,\uparrow) & (2,\uparrow) & (5,\uparrow) & (0,-) \\ \hline (3,\downarrow) & (0,-) & (0,-) & (0,-) \\ \hline \end{array} \begin{array}{c} \text{Rest}_{12} \\ \downarrow \end{array} \{1, 4, 6, 8\} \end{array} \right)$$

$$f_{\text{field},2}(s_{12}) = (2, 3)$$

$$f_{\text{piece},2}((A_{12}, \text{Rest}_{12}), 2, 3) = (4, \downarrow)$$

Search Control 2

$$S_{13} = \left(\begin{array}{c|c|c|c} & \overset{A_{13}}{\downarrow} & & \overset{\text{Rest}_{13}}{\downarrow} \\ \hline (7, \uparrow) & (2, \uparrow) & (5, \uparrow) & (0, -) \\ \hline (3, \downarrow) & (0, -) & (4, \downarrow) & (0, -) \\ \hline \end{array} \right), \{1, 6, 8\}$$

$f_{\text{field},2}(s_{13}) = (1, 4)$ – at this point we could go with any of the 3

$$f_{\text{piece},2}((A_{13}, \text{Rest}_{13}), 1, 4) = (6, \uparrow)$$

Search Control 2

$$S_{14} = \left(\begin{array}{|c|c|c|c|} \hline (7, \uparrow) & (2, \uparrow) & (5, \uparrow) & (6, \uparrow) \\ \hline (3, \downarrow) & (0, -) & (4, \downarrow) & (0, -) \\ \hline \end{array} \right)_{\{1, 8\}}$$

A_{14} $Rest_{14}$
 $\downarrow$ $\downarrow$

$$f_{\text{field},2}(s_{14}) = (2, 4)$$

$$f_{\text{piece},2}((A_{14}, Rest_{14}), 2, 4) = (1, \downarrow)$$

Search Control 2

$$S_{15} = \left(\begin{array}{c|c|c|c} & \overset{A_{15}}{\downarrow} & & \\ \hline (7, \uparrow) & (2, \uparrow) & (5, \uparrow) & (6, \uparrow) \\ \hline (3, \downarrow) & (0, -) & (4, \downarrow) & (1, \downarrow) \\ \hline \end{array} \right\{8\}$$

$\overset{\text{Rest}_{15}}{\downarrow}$

$$f_{\text{field},2}(s_{15}) = (2, 2)$$

$$f_{\text{piece},2}((A_{15}, \text{Rest}_{15}), 2, 4) = (8, \downarrow)$$

Search Control 2

A_{16}
↓

$Rest_{16}$
↓

$$S_{16} = \left(\begin{array}{|c|c|c|c|} \hline (7,\uparrow) & (2,\uparrow) & (5,\uparrow) & (6,\uparrow) \\ \hline (3,\downarrow) & (8,\downarrow) & (4,\downarrow) & (1,\downarrow) \\ \hline \end{array} \right\}$$

$$G_{puzzle1}(s_{16}) = \text{yes}$$

$$\text{Since } |Rest_{16}| = 0$$

Search Control 3

- Change our definition of a 'fitting' piece
- Change f_{field} to ignore positions that put straight sides facing inside the matrix
- Change f_{piece} to ignore pieces/orientations that put straight sides facing inside the matrix

Search Control 3

$$S_0 = \left(\begin{array}{c} \textcolor{violet}{A_0} \downarrow \\ \begin{array}{|c|c|c|c|} \hline (0,-) & (0,-) & (0,-) & (0,-) \\ \hline (0,-) & (0,-) & (0,-) & (0,-) \\ \hline \end{array} , \textcolor{violet}{Rest_0} \downarrow \{1, 2, 3, 4, 5, 6, 7, 8\} \end{array} \right)$$

$$f_{\text{field},3}(s_0) = (1, 1)$$

$$f_{\text{piece},3}((A_0, \text{Rest}_0), 1, 1) = (1, \uparrow), (3, \rightarrow) \\ (6, \rightarrow), (7, \uparrow)$$

Search Control 3

$$S_1 = \left(\begin{array}{c|c|c|c} & \textcolor{violet}{A_1} \downarrow & & \textcolor{violet}{Rest_1} \downarrow \\ \hline (7, \uparrow) & (0, -) & (0, -) & (0, -) \\ \hline (0, -) & (0, -) & (0, -) & (0, -) \end{array} , \{1, 2, 3, 4, 5, 6, 8\} \right)$$

$$f_{\text{field},3}(s_1) = (2, 1)$$

$$f_{\text{piece},3}((A_1, \text{Rest}_1), 2, 1) = (3, \downarrow)$$

Search Control 3

$$S_2 = \left(\begin{array}{c|c|c|c} & \textcolor{violet}{A_2} \downarrow & & \textcolor{violet}{Rest_2} \downarrow \\ \hline (7, \uparrow) & (0, -) & (0, -) & (0, -) \\ \hline (3, \downarrow) & (0, -) & (0, -) & (0, -) \end{array} \right) \{1, 2, 4, 5, 6, 8\}$$

$$f_{\text{field},3}(s_2) = (1, 2)$$

$$f_{\text{piece},3}((A_2, \text{Rest}_2), 1, 2) = (2, \uparrow), (5, \uparrow)$$

Search Control 3

$$S_3 = \left(\begin{array}{c} \textcolor{violet}{A}_3 \\ \downarrow \end{array} \begin{array}{|c|c|c|c|} \hline (7,\uparrow) & (5,\uparrow) & (0,-) & (0,-) \\ \hline (3,\downarrow) & (0,-) & (0,-) & (0,-) \\ \hline \end{array} \begin{array}{c} \textcolor{violet}{Rest}_3 \\ \downarrow \end{array} \{1, 2, 4, 6, 8\} \right)$$

Can't put down $(5,\uparrow)$ because it no longer 'fits'

$$f_{\text{remove},3}((A_3, \textcolor{violet}{Rest}_3)) = (1, 2) \text{ or } (2, 1)$$

Search Control 3

$$S_4 = \left(\begin{array}{c|c|c|c} & \overset{A_4}{\downarrow} & & \overset{Rest_4}{\downarrow} \\ \hline (7, \uparrow) & (0, -) & (0, -) & (0, -) \\ \hline (3, \downarrow) & (0, -) & (0, -) & (0, -) \end{array} , \{1, 2, 4, 5, 6, 8\} \right)$$

$$f_{\text{field},3}(s_4) = (1, 2)$$

$$f_{\text{piece},3}((A_4, Rest_4), 1, 2) = (2, \uparrow)$$

Search Control 3

$$S_5 = \left(\begin{array}{c|c|c|c} & \textcolor{violet}{A_5} \downarrow & & \textcolor{violet}{Rest_5} \downarrow \\ \hline (7, \uparrow) & (2, \uparrow) & (0, -) & (0, -) \\ \hline (3, \downarrow) & (0, -) & (0, -) & (0, -) \end{array} \right), \{1, 4, 5, 6, 8\}$$

$$f_{\text{field},3}(s_5) = (1, 3)$$

$$f_{\text{piece},3}((A_5, Rest_5), 1, 3) = (5, \uparrow)$$

Search Control 3

$$S_6 = \left(\begin{array}{c|c|c|c} & \overset{A_6}{\downarrow} & & \overset{Rest_6}{\downarrow} \\ \hline (7, \uparrow) & (2, \uparrow) & (5, \uparrow) & (0, -) \\ \hline (3, \downarrow) & (0, -) & (0, -) & (0, -) \end{array} \right) \{1, 4, 6, 8\}$$

$$f_{\text{field},3}(s_6) = (2, 3)$$

$$f_{\text{piece},3}((A_6, Rest_6), 2, 3) = (4, \downarrow)$$

Search Control 3

$$S_7 = \left(\begin{array}{|c|c|c|c|} \hline (7,\uparrow) & (2,\uparrow) & (5,\uparrow) & (0,-) \\ \hline (3,\downarrow) & (0,-) & (4,\downarrow) & (0,-) \\ \hline \end{array} \right)_{\{1, 6, 8\}}$$

A_7 $Rest_7$
 $\downarrow$ $\downarrow$

$f_{\text{field},3}(s_7) = (1,4)$ – arbitrarily

$f_{\text{piece},3}((A_7, Rest_7), 1, 4) = (6,\uparrow)$

Search Control 3

$$S_8 = \left(\begin{array}{|c|c|c|c|} \hline (7,\uparrow) & (2,\uparrow) & (5,\uparrow) & (6,\uparrow) \\ \hline (3,\downarrow) & (0,-) & (4,\downarrow) & (0,-) \\ \hline \end{array} \right)_{\{1, 8\}}$$

A_8 $Rest_8$
 $\downarrow$ $\downarrow$

$f_{\text{field},3}(s_8) = (2,4)$ – arbitrarily

$f_{\text{piece},3}((A_8, Rest_8), 2, 4) = (1,\downarrow)$

Search Control 3

$$S_9 = \left(\begin{array}{|c|c|c|c|} \hline (7,\uparrow) & (2,\uparrow) & (5,\uparrow) & (6,\uparrow) \\ \hline (3,\downarrow) & (0,-) & (4,\downarrow) & (1,\downarrow) \\ \hline \end{array} \right)_{\{8\}}$$

A_9 $Rest_9$
 $\downarrow$ $\downarrow$

$$f_{\text{field},3}(s_9) = (2,2)$$

$$f_{\text{piece},3}((A_9, Rest_9), 2, 2) = (8,\downarrow)$$

Search Control 3

$$S_{10} = \left(\begin{array}{|c|c|c|c|} \hline (7,\uparrow) & (2,\uparrow) & (5,\uparrow) & (6,\uparrow) \\ \hline (3,\downarrow) & (8,\downarrow) & (4,\downarrow) & (1,\downarrow) \\ \hline \end{array} \right\}$$

A_{10} ↓

$Rest_{10}$ ↓

$$G_{puzzle1}(s_{10}) = \text{yes}$$

$$\text{Since } |Rest_{10}| = 0$$