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Min-Cost Multicast of Selfish Information Flows

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Talk Outline

- Min-Cost Multicast
- Selfish Flows
- Uncapacitated Networks
- Capacitated Networks
- Algorithms
Min-Cost Network Flow

capacity, cost
Min-Cost Multicast

unit edge cost and capacity, target multicast rate = 1

Algorithm: [Lun et al., INFOCOM 2005]
Selfish Information Flows

- Always takes cheapest path
- Nash Equilibrium
- Price of Anarchy
- Price of Stability
Enforcing Min-Cost Multicast

Solution Space:

- Flow Routing
- Cost Allocation

Obvious Goals:

- Stability
- Optimality
- Budget Balance
- Fairness
Anything **Local** Will Not Work

- Equal Share
- Proportional Share
- Shapley Value
Inspirations

• Try shadow prices [Fleischer, Jain, Mahdian, FOCS 2004]

Which shadow prices?

• Conceptual flow merging [LLJL, INFOCOM 2005] $\leftrightarrow$ tree path merging
Uncapacitated Networks

Minimize \[ \sum_e w(e)f(e) \]

Subject to:
\[
\begin{align*}
\sum_{p \in \mathcal{P}_i} f(p) &= d \quad \forall i \quad \leftrightarrow x_i \\
f(e) &\geq \sum_{p \in \mathcal{P}_i : e \in p} f(p) \quad \forall i, \forall e \quad \leftrightarrow y_i(e)
\end{align*}
\]
\[ f(e), f(p) \geq 0 \quad \forall e, \forall p \]

Maximize \[ \sum_i x_i d \]

Subject to:
\[
\begin{align*}
\sum_i y_i(e) &\leq w(e) \quad \forall e \quad \leftrightarrow f(e) \\
x_i &\leq \sum_{e \in p} y_i(e) \quad \forall i, \forall p \in \mathcal{P}_i \quad \leftrightarrow f(p)
\end{align*}
\]
\[ x_i \leftrightarrow 0; y_i(e) \geq 0 \quad \forall i, \forall e \]
Cost Allocation Based on Shadow Prices

Theorem 1. If $f^*$ and $(x^*, y^*)$ are a pair of optimal solutions to the primal and dual min-cost multicast LPs respectively, then $y^*$ enforces $f^*$.

- stability ✓
- optimality ✓
- budget balance ✓
- fairness ✓

Proof based on: LP duality, complimentary slackness
Capacitated Networks: Cannot Ignore Capacity Bounds

(A) S

M

2 <1,1>

2 <2,0>

T_1

N

10 <5,5>

40 <20,20>

T_2

30 <15,15>

(B) S

M

2 <1,1>

2 <2,0>

T_1

N

10 <10,0>

40 <0,40>

T_2

30 <5,25>
Introducing edge taxes in accordance to “tightness” of bandwidth supply
Capacitated Networks

Minimize \( \sum_e w(e) f(e) \)

Subject to:
\[
\begin{align*}
\sum_{p \in \mathcal{P}_i} f(p) &= d \quad \forall i \\
&\leftrightarrow x_i \\
f(e) &\geq \sum_{p \in \mathcal{P}_i : e \in p} f(p) \quad \forall i, \forall e \leftrightarrow y_i(e) \\
f(e) &\leq c(e) \quad \forall e \leftrightarrow t(e) \\
f(e), f(p) &\geq 0 \quad \forall e, \forall p
\end{align*}
\]

Maximize \( \sum_i x_i d - \sum_e c(e) t(e) \)

Subject to:
\[
\begin{align*}
\sum_i y_i(e) &\leq w(e) + t(e) \quad \forall e \leftrightarrow f(e) \\
x_i &\leq \sum_{e \in p} y_i(e) \quad \forall i, \forall p \in \mathcal{P}_i \leftrightarrow f(p) \\
x_i &\leftrightarrow 0; y_i(e), t(e) \geq 0 \quad \forall i, \forall e
\end{align*}
\]
Cost Allocation with Edge Taxes

Theorem 2. If $f^*$ and $(x^*, y^*, t^*)$ are optimal solutions to the primal and dual min-cost multicast LPs respectively, then $(y^*, t^*)$ strictly enforces $f^*$.

- stability ✓
- optimality ✓
- budget balance ✓ (with $w + t$ as cost)
- fairness ✓
- strict enforcement ✓
Theorem 3. In a capacitated network, every optimal multicast flow $f^*$ can be enforced by a cost allocation scheme $y'$.

- Taxes only appear on “tight” edges
- Proportional return
Algorithm Design

- (Simultaneously) Computing $f, y, t$
- Lagrange Relaxation
- Subgradient Optimization
Some Related Research

- Enforcing multicommodity flows, [Fleischer et al. 2004][Karakostas et al. 2004]
- Strictly convex cost, [Bhadra et al. 2006]
- Shapley value based cost sharing, [Feigenbaum et al. 2001]
- Strategyproof Multicast, [Wang et al. 2004]
THANKS